Published by Institute of Physics Publishing for SISSA

Received: *September 20, 2007*
Accepted: *November 26, 2007*
Published: *December 4, 2007*

Near-horizon geometries of supersymmetric AdS_5 black holes

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Abstract: We provide a classification of near-horizon geometries of supersymmetric, asymptotically anti-de Sitter, black holes of five-dimensional $\mathrm{U}(1)^3$-gauged supergravity which admit two rotational symmetries. We find three possibilities: a topologically spherical horizon, an $S^1 \times S^2$ horizon and a toroidal horizon. The near-horizon geometry of the topologically spherical case turns out to be that of the most general known supersymmetric, asymptotically anti-de Sitter, black hole of $\mathrm{U}(1)^3$-gauged supergravity. The other two cases have constant scalars and only exist in particular regions of this moduli space — in particular they do not exist within minimal gauged supergravity. We also find a solution corresponding to the near-horizon geometry of a three-charge supersymmetric black ring held in equilibrium by a conical singularity; when lifted to type IIB supergravity this solution can be made regular, resulting in a discrete family of warped AdS_3 geometries. Analogous results are presented in $\mathrm{U}(1)^n$ gauged supergravity.

Keywords: Black Holes in String Theory, Black Holes, Supergravity Models.

Contents

1. Introduction

Supersymmetric, asymptotically $AdS_5 \times S^5$, black holes have only been known for a few years [1 − 4]. Their relevance stems from the AdS/CFT correspondence [5], which implies such black holes should be dual to 1/16 BPS states of $\mathcal{N} = 4$ SU(N) Yang-Mills theory on $R \times S^3$. Such states are, generically, specified by five quantum numbers: the SO(4) spins J_1, J_2 and the SO(6) R-charges Q_1, Q_2, Q_3. However, the most general *known* black hole of this kind has a constraint relating these leaving only four independent conserved charges [4]. Indeed, this is the most general known regular,[1] asymptotically $AdS_5 \times S^5$, 1/16 BPS,

[1] Here we allow for solutions regular on and outside an event horizon, which typically possess a singularity behind the horizon and thus are not globally regular.

solution of type IIB supergravity. Therefore, before attempting to derive the Hawking-Bekenstein entropy from a microscopic counting in the field theory, this discrepancy in the number of 1/16 BPS states between the two dual pictures needs to be understood [6].

There are three ways this potential contradiction could get resolved. One possibility is that we already know the most general black hole and due to finite coupling effects in the CFT only a four parameter subset contribute to the $O(N^2)$ entropy at large N (see e.g. [7]). A second possibility is that there is a sufficient number of asymptotically $AdS_5 \times S^5$, 1/16 BPS, solitons[2] to account for the missing states, although we note that no examples of such solutions are known.[3] The final possibility, which we shall explore in this paper, is that there is a more general family of black holes. This latter possibility can be realised in a number of ways, raising the following questions:

1. The known solutions are all solutions to five dimensional gauged supergravity, which is a consistent truncation of type IIB supergravity on S^5;[4] do asymptotically $AdS_5 \times S^5$ black holes exist within type IIB that are truly ten dimensional? — i.e. depend on the higher spherical harmonics on S^5.

2. The known solutions are solutions to a truncation of the maximal 5d SO(6)-gauged supergravity where one only keeps the maximal abelian subgroup $\mathrm{U}(1)^3$ (with two scalars). Are there 5d black holes with extra scalars and/or non-abelian gauge fields turned on?

3. The known class of black holes possess topologically spherical horizons (in 5d language); are there black holes with more exotic horizon topology, such as black rings?

4. Is there a more general black hole in the 5d $\mathrm{U}(1)^3$ gauged supergravity than the currently known one?

In this paper we will only be concerned with black hole solutions of 5d $\mathrm{U}(1)^3$ gauged supergravity[5] and therefore can only address questions 4 and 3 (partially). Black hole uniqueness theorems have not been established for asymptotically AdS spacetimes, even in four dimensions. Further, it is known that these theorems fail for asymptotically flat spacetimes in five dimensions [8]. It is therefore unclear how many AdS_5 black hole solutions one should expect to exist for given conserved charges.[6] However, we are concerned with

[2]By this we mean smooth, strictly stationary solutions — therefore horizonless.

[3]These would be analogues of the 1/2 BPS LLM geometries [11]. However, even the 1/4 and 1/8 BPS cases are far from being understood [12]. We should also note that it is not clear that for given conserved charges black holes and solitons should be counted on the same footing.

[4]It is only known how to perform the reduction from the subsector of IIB which keeps the metric and self-dual five form [13].

[5]By this we mean minimal gauged supergravity coupled to two abelian vector multiplets.

[6]In particular it is not clear whether they should be uniquely specified by their conserved charges, even for the case of spherical topology. Indeed, if one assumes this is the case, then a non-extremal black hole would be parameterised by 6 conserved charges: $M, J_1, J_2, Q_1, Q_2, Q_3$. In general the BPS limit will reduce the number of parameters of a black hole by two: this is because extremality and supersymmetry do not lead to the same constraint. This leaves one with a 4 parameter black hole, presumably the one which is already known [4].

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supersymmetric solutions. There exist systematic methods that help one construct such solutions [9, 2]. Unfortunately, guesswork is still required (i.e. choice of a Kähler metric on the base manifold), thus leaving the task of a full classification of supersymmetric AdS_5 black holes out of reach for the moment.

Since supersymmetric black holes are extremal they admit a near-horizon limit. This limit allows one to focus on a region in the neighbourhood of the horizon in such a way that the limit is a solution to the same theory the black hole is. Thus a more modest goal presents itself: a classification of the near-horizon geometries of asymptotically AdS_5 black holes.

Recently, we performed such an analysis within minimal gauged supergravity [10] under the assumption that the black hole admits two rotational symmetries. It was shown the near-horizon geometry of an asymptotically AdS_5 supersymmetric black hole, admitting two rotational symmetries must have a horizon of spherical topology and is given by the near-horizon limit of the black holes of [3]. In particular this ruled out the existence of supersymmetric black rings with these symmetries.

The purpose of this paper is to generalise this analysis to the $\mathrm{U}(1)^3$ gauged supergravity. Qualitatively, our results differ to those we found in minimal supergravity. In $\mathrm{U}(1)^3$ gauged supergravity we find that the most general topologically spherical supersymmetric black hole, with two rotational symmetries, is the near-horizon geometry of the solution found in [4] (this result is analogous to the one in minimal supergravity). However, we find that other topologies with these symmetries are also allowed: namely $S^1 \times S^2$ and T^3 . The corresponding near-horizon solutions are $AdS_3 \times S^2$ and $AdS_3 \times T^2$ respectively and must have constant scalars.[7] These solutions only exist in a particular region of the scalar moduli space which does not include the minimal theory. Further, the $S^1 \times S^2$ case is only a three parameter family of solutions, whereas the near-horizon of the supersymmetric black ring in ungauged supergravity [21] is a four parameter family. Nevertheless this means we are not able to rule out the existence of supersymmetric anti-de Sitter black rings in $\mathrm{U}(1)^3$-gauged supergravity. However, we should emphasise, that the existence of a near-horizon geometry does *not* imply that a corresponding black hole solution with prescribed asymptotics (i.e. in our case global AdS_5) exists.

The above three cases are the only possible regular near-horizon geometries with two rotational symmetries. However as in our analysis of minimal supergravity [10] we also find a solution describing the near horizon of an unbalanced black ring supported by a conical singularity. It consists of a warped product of AdS_3 and a singular S^2 and reduces to the near-horizon of the black ring in ungauged supergravity. When lifted to type IIB supergravity we show that this solution can be made regular, resulting in a discrete set of regular warped AdS_3 geometries.

This paper is organized as follows. In the section 2, we summarise the main results of our analysis for $\mathrm{U}(1)^3$ supergravity. The remainder of the paper will be dedicated to the derivation of these results. We found it convenient to work in the more general $\mathrm{U}(1)^n$ gauged supergravity. In section 3 we derive the conditions imposed by supersymmetry and

[7]Such product geometries in $\mathrm{U}(1)^3$-gauged supergravity have in fact been noticed before [20].

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the equations of motion in the near-horizon limit. In section 4 we determine all the possible near-horizon geometries of a supersymmetric AdS black hole with $\mathrm{U}(1)^2$ spatial isometry (i.e. two rotational symmetries) and perform a detailed global analysis of these geometries. We conclude with a Discussion. Some details of the analysis are given in the appendix.

2. Main results

In this section we explicitly state the main results of this paper. This section is intended to be a self-contained account without derivations.

The action for the bosonic sector of five-dimensional $\mathrm{U}(1)^3$-gauged supergravity is:

$$S = \frac{1}{16\pi G}\int d^5x\Big(\sqrt{g}\Big[R+4g^2\sum_{I=1}^{3}(X^I)^{-1}-\frac{1}{2}\sum_{I=1}^{3}(X^I)^{-2}(\partial X^I)^2-\frac{1}{4}\sum_{I=1}^{3}(X^I)^{-2}F^I\cdot F^I\Big] \\ -F^1\wedge F^2\wedge A^3\Big) \tag{2.1}$$

where $F^I = dA^I$ and the scalars satisfy the constraint $X^1X^2X^3 = 1$ (g is the gauge coupling). Note that minimal gauged supergravity is the following truncation of the above theory: $X^I = 1$ and $A^I = A$. Consider a supersymmetric, asymptotically AdS, black hole solution of the above theory with isometry group $R\times \mathrm{U}(1)\times \mathrm{U}(1)$ (this corresponds to time translational symmetries and spatial rotational symmetries). Spatial sections of the horizon of such a black hole must be: topologically spherical, $S^1\times S^2$ or toroidal. Below we list the most general near-horizon solutions corresponding to these cases.

Topologically spherical horizon. The near-horizon solution in this case is always non-static. The main assumption of our analysis is that the solutions possess $\mathrm{U}(1)^2$ rotational symmetry. However, as is typical of rotating solutions in 5d, we find there is a special case for which one has a symmetry enhancement of the rotational group to $\mathrm{SU}(2)\times\mathrm{U}(1)$. Generically though we find one only[8] has the $\mathrm{U}(1)^2$. As a byproduct of our analysis we have found efficient coordinate systems which describe these two cases separately.

The more symmetric and hence simplest case can be written as

$$ds^2_{NH} = -(\Delta^2+g^2X^2)r^2dv^2+2dvdr+\left[\frac{\Delta}{\Delta^2+g^2\lambda}(d\phi+\cos\theta d\psi)-gXrdv\right]^2 \\ +\frac{1}{\Delta^2+g^2\lambda}(d\theta^2+\sin^2\theta d\psi^2) \tag{2.2}$$

$$A^I = \Delta X^I rdv-\frac{gX^I(X-2X^I)}{\Delta^2+g^2\lambda}(d\phi+\cos\theta d\psi) \tag{2.3}$$

where the scalars X^I are constant and we have defined the constants:

$$X \equiv X^1+X^2+X^3 \tag{2.4}$$

$$\lambda \equiv (X^1)^2+(X^2)^2+(X^3)^2-2X^1X^2-2X^1X^3-2X^2X^3. \tag{2.5}$$

[8] In contrast, in the ungauged theory one *must* have $\mathrm{SU}(2)\times\mathrm{U}(1)$ rotational symmetry (this is the near-horizon of the BMPV black hole) [14, 18].

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The solution is parameterized by the constants (X^I, Δ) subject to the constraints $\Delta > 0$ and $\Delta^2 + g^2\lambda > 0$ (and of course $X^1X^2X^3 = 1$) and is therefore a three parameter family. The coordinate ranges are: $0 \leq \psi < 2\pi$, $0 \leq \phi < 4\pi$, $0 \leq \theta \leq \pi$. The horizon is located at $r = 0$ and on spatial sections is a homogeneously squashed S^3. The near-horizon geometry in this case is a homogeneous space and is a fibration of AdS_2 over the homogeneously squashed S^3 with symmetry group $\mathrm{SO}(2,1) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. This solution turns out to be the near-horizon limit of the black holes found in [2],[9] which are characterised by $J_1 = J_2$ and are special cases of the more general solution of [4].

The generic case is more complicated. In this case the near-horizon solution can be written as:

$$\begin{aligned} ds^2_{NH} &= H(x)^{1/3}\left[-C^2R^2dv^2 + 2dvdR + \left(C^2 - \frac{\Delta_0^2}{H(x)}\right)\left(dx^1 + \omega(x)dx^2 + Rdv\right)^2\right] \\ &\quad + \frac{H(x)^{1/3}dx^2}{4g^2P(x)} + \frac{4g^2H(x)^{1/3}P(x)(dx^2)^2}{C^2H(x) - \Delta_0^2}, \end{aligned} \tag{2.6}$$

$$X^I = \frac{H(x)^{1/3}}{x + 3K_I}, \qquad A^I = \frac{X^I}{H(x)^{1/3}}\left[\Delta_0(Rdv + dx^1) + (x - \alpha_0)dx^2\right] \tag{2.7}$$

where $C^2, \alpha_0, \Delta_0, K_I$ are constants such that $C, \Delta_0 > 0$ and $K_1 + K_2 + K_3 = 0$ and the functions are defined by

$$H(x) = (x + 3K_1)(x + 3K_2)(x + 3K_3), \qquad P(x) = H(x) - \frac{C^2}{4g^2}(x - \alpha_0)^2 - \frac{\Delta_0^2}{C^2}, \tag{2.8}$$

$$\omega(x) = \frac{\Delta_0(\alpha_0 - x)}{C^2H(x) - \Delta_0^2}. \tag{2.9}$$

Due to a scaling symmetry of the solution it turns out one parameter is trivial (for instance one can set C to any desired value); therefore this is a 4-parameter family of solutions. The coordinate ranges are $x_1 \leq x \leq x_2$ where $0 < x_1 < x_2 < x_3$ are the three roots of the cubic $P(x)$, such that $x_1 + 3K_I > 0$, and the coordinates defined by $\partial/\partial\phi_i \propto \omega(x_i)\partial/\partial x^1 - \partial/\partial x^2$ $(i = 1, 2)$ are 2π-periodic. The horizon is located at $R = 0$ and spatial sections of this must possess S^3 topology with $\partial/\partial\phi_1$ vanishing at the pole $x = x_1$ and $\partial/\partial\phi_2$ vanishing at the pole $x = x_2$ — these are the generators of the $\mathrm{U}(1)^2$ rotational symmetries. In this case the near-horizon solution is a fibration of AdS_2 over a (non-homogeneously) squashed S^3 with an $\mathrm{SO}(2,1) \times \mathrm{U}(1)^2$ symmetry group and is cohomogeneity-1. It turns out it is 1/2 BPS within $\mathrm{U}(1)^3$-gauged supergravity.[10] This solution turns out to be the near-horizon limit of the black holes found in [4] which have $J_1 \neq J_2$.

Therefore the most general near-horizon solution with a topologically spherical horizon and two rotational symmetries, does in fact turn out to be the near-horizon limit of the 4-parameter black holes of [4]. The coordinates of that paper, while not allowing such compact expressions as above, do in fact cover both of the above cases.

The $\mathrm{SO}(2,1)$ symmetry of the above near-horizon solutions is guaranteed from the general results of [15].

[9] The near-horizon geometries of these black holes have been previously studied in [22–24].

[10] This follows from the fact that it is related by an analytic continuation (of the form considered in [15]) to the Sabra-Klemm time machines [16].

$S^1 \times S^2$ horizon. The near-horizon solution is static and takes the form:

$$\begin{aligned} ds^2_{NH} &= 2dvdr - 2gXrdvdz + dz^2 + \frac{1}{g^2\lambda}(d\theta^2 + \sin^2\theta d\psi^2), \\ A^I &= -\frac{1}{g\lambda}X^I(X - 2X^I)\cos\theta d\psi \end{aligned} \tag{2.10}$$

where the scalars X^I are constants and X, λ are the constants defined in (2.4) and (2.5). The solution is parameterised by the constants (L, X^I), where L is the period of z (which can be arbitrary), subject to the constraints $\lambda > 0$ (and of course $X^1X^2X^3 = 1$) and therefore is a three parameter family. Note that the constraint $\lambda > 0$ can be satisfied by $\sqrt{X^1} + \sqrt{X^2} < \sqrt{X^3}$ (or permutations of 123) for example. Also observe that this solution does not exist in minimal gauged supergravity because in this case $X^I = 1$ and therefore $\lambda = -3$. The near-horizon geometry in this case is $AdS_3 \times S^2$. The horizon is located at $r = 0$ and spatial sections are $S^1 \times S^2$ equipped with a direct product metric with a round S^2. This solution is 1/2 BPS [20].

Toroidal horizon. The near-horizon solution is static and given by

$$ds^2_{NH} = 2dvdr - 2gXrdvdz + dz^2 + dx^2 + dy^2, \quad A^I = \frac{gX^I}{2}(X - 2X^I)(xdy - ydx) \tag{2.11}$$

where the scalars X^I are constants which satisfy $\lambda = 0$, and X, λ are defined by (2.4) and (2.5). This can be achieved by taking $\sqrt{X^1} + \sqrt{X^2} = \sqrt{X^3}$ (or permutations of 123) for example. This solution is not allowed in minimal gauged supergravity. The near-horizon geometry in this case is $AdS_3 \times T^2$. The horizon is at $r = 0$ and spatial sections are of toroidal topology equipped with a flat metric. This solution is 1/2 BPS [20].

The above three cases are the only possible *regular* near-horizon geometries with two rotational symmetries. Therefore, in particular, if an anti-de Sitter black ring with two rotational symmetries exists in this theory, it *must* have the near-horizon geometry given by (2.10). We will consider this possibility in the Discussion.

Curiously, though, within our analysis we did find another $S^1 \times S^2$ case, where the S^2 necessarily possesses a conical singularity at one of its poles. The solution in this case is locally given by the $\Delta_0 \to 0$ limit of the topologically spherical case (2.6) and is also a four parameter family (despite losing the parameter Δ_0 one aquires a fourth parameter from the period of the S^1). Note that the $g \to 0$ limit of this solution reduces to the non-singular four-parameter near-horizon geometry of the asymptotically flat supersymmetric black ring of ungauged supergravity, $AdS_3 \times S^2$ [21]. This could therefore correspond to the near-horizon limit of an *unbalanced* supersymmetric anti-de Sitter black ring. For $g > 0$ this singular near-horizon geometry can in fact be made regular when oxidised to IIB supergravity; however the resulting geometry can no longer be viewed as a solution of 5d supergravity.

This completes the list of all possible near-horizon limits of supersymmetric AdS_5 black holes with symmetry $R \times \mathrm{U}(1) \times \mathrm{U}(1)$, in $\mathrm{U}(1)^3$-gauged supergravity. In the subsequent sections we perform a systematic analysis where we prove these are the only possibilities.

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We actually found it convenient to work in the more general $U(1)^n$ supergravity and thus we have obtained analogues of the above results in this theory.

3. Supersymmetric near-horizon geometries

3.1 Gauged supergravity

We shall consider the theory of five dimensional $\mathcal{N} = 1$ gauged supergravity coupled to $n-1$ abelian vector multiplets following the conventions of [2]. The bosonic sector of this theory consists of the graviton, n vectors A^I and $n-1$ real scalars. The latter can be replaced with n real scalars X^I subject to a constraint

$$\frac{1}{6}C_{IJK}X^I X^J X^K = 1, \tag{3.1}$$

where C_{IJK} are a set of real constants symmetric under permutations of (IJK). Indices $I, J, K, \ldots$ run from 1 to n. It is convenient to define

$$X_I \equiv \frac{1}{6}C_{IJK}X^J X^K. \tag{3.2}$$

The action is[11]

$$S = \frac{1}{16\pi G}\int\left(R_5 \star 1 - Q_{IJ}F^I \wedge \star F^J - Q_{IJ}dX^I \wedge \star dX^J - \frac{1}{6}C_{IJK}F^I \wedge F^J \wedge A^K + 2\chi^2 \mathcal{V} \star 1\right), \tag{3.3}$$

where $F^I \equiv dA^I$ and

$$Q_{IJ} \equiv \frac{9}{2}X_I X_J - \frac{1}{2}C_{IJK}X^K. \tag{3.4}$$

For simplicity, we shall assume that the scalars parameterize a symmetric space, which is equivalent to the condition

$$C_{IJK}C_{J'(LM}C_{PQ)K'}\delta^{JJ'}\delta^{KK'} = \frac{4}{3}\delta_{I(L}C_{MPQ)}. \tag{3.5}$$

This condition ensures that the matrix Q_{IJ} is invertible, with inverse

$$Q^{IJ} = 2X^I X^J - 6C^{IJK}X_K, \tag{3.6}$$

where

$$C^{IJK} \equiv C_{IJK}. \tag{3.7}$$

We then have

$$X^I = \frac{9}{2}C^{IJK}X_J X_K. \tag{3.8}$$

The scalar potential is

$$\mathcal{V} = 27C^{IJK}V_I V_J X_K \tag{3.9}$$

where V_I are a set of constants. Without loss of generality we will assume $X^I, V_I > 0$ and $C_{IJK} \geq 0$, so $\mathcal{V} > 0$. It was shown in [2] that the unique maximally supersymmetric solution

[11] We use a positive signature metric.

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of this theory is AdS_5 with vanishing vectors and constant scalars given by $X_I = \bar{X}_I \equiv V_I/\xi$, where $\xi^3 = \frac{9}{2} C^{IJK} V_I V_J V_K$, with the radius of AdS_5 given by $g^{-1} \equiv (\xi\chi)^{-1}$. The above theory can be consistently truncated to minimal gauged supergravity as follows: $A^I = \bar{X}^I \mathcal{A}$ and $X^I = \bar{X}^I$.

In this paper, we are interested in a particular $\mathrm{U}(1)^3$ gauged supergravity that arises upon compactification of Type IIB supergravity on S^5. In the above language, this theory has $n = 3$, $C_{IJK} = 1$ if (IJK) is a permutation of (123) and $C_{IJK} = 0$ otherwise, and $\bar{X}^I = 1$ (so $\bar{X}_I = 1/3$). In this case the action reduces to (2.1).

Supersymmetric solutions. The general nature of supersymmetric solutions of this theory was deduced in [2] following closely the corresponding analysis for the minimal theory given in [9]. We will briefly summarise some of the results of this analysis. Given a supercovariantly constant spinor ϵ, one can construct a real scalar $f \sim \bar{\epsilon}\epsilon$ and a real vector $V^\mu \sim \bar{\epsilon}\gamma^\mu\epsilon$ and three real two forms $J^i_{\mu\nu} \sim \bar{\epsilon}\gamma_{\mu\nu}\epsilon$ where $i = 1, 2, 3$. These obey $V^2 = -f^2$, so V is timelike or null, and it turns out that V is always Killing. Also note:

$$dJ^i = 3\chi\epsilon^{1ij} V_I \left(A^I \wedge J^j + X^I \star_5 J^j\right) \tag{3.10}$$

which we will need later. There are two cases: a "null" case, in which V is globally null and a "timelike" case in which V is timelike in some region $\mathcal{U}$ of spacetime. The former case was treated in [9, 17] and does not concern us here because such solutions cannot describe black holes.

In the timelike case, we can, without loss of generality, assume that $f > 0$ in $\mathcal{U}$, and introduce local coordinates so that the metric takes the form

$$ds^2 = -f^2 (dt + \omega)^2 + f^{-1} h_{mn} dx^m dx^n, \tag{3.11}$$

with $V = \partial/\partial t$, h_{mn} is a metric on a 4-dimensional Riemannian "base space" $\mathcal{B}$ and ω a 1-form on $\mathcal{B}$. We choose the orientation on $\mathcal{B}$ so that $(dt + \omega) \wedge \eta_4$ is positively oriented in space-time, where η_4 is the volume form on $\mathcal{B}$. Fierz identities imply the J^i are anti-self dual two-forms defined on $\mathcal{B}$ that satisfy the algebra of the unit quaternions — hence $\mathcal{B}$ admits an almost hyper-Kähler structure. Supersymmetry then implies that the base space is Kähler with Kähler form J^1. Necessary and sufficient conditions for the existence of a supercovariantly constant spinor, in the timelike class, are derived in [2] and all take the form of equations defined on $\mathcal{B}$. We will not record all those conditions here, however we note that supersymmetry implies [2]

$$F^I = d\left[X^I f(dt + \omega)\right] + \Theta^I - 9\chi f^{-1} C^{IJK} V_J X_K J^1, \tag{3.12}$$

and

$$X_I \Theta^I = -\frac{2}{3} G^+, \tag{3.13}$$

where Θ^I are self-dual 2-forms on $\mathcal{B}$ and

$$G^\pm = \frac{1}{2} f \left(d\omega \pm \star_4 d\omega\right), \tag{3.14}$$

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where $\star_4$ is the Hodge dual on $\mathcal{B}$. The field equations of the theory are all satisfied once we impose the equations of motion for the Maxwell fields [2], i.e., the Bianchi identities

$$dF^I = 0, \tag{3.15}$$

and the Maxwell equations,

$$d\left(Q_{IJ} \star F^J\right) = -\frac{1}{4} C_{IJK} F^J \wedge F^K. \tag{3.16}$$

One can substitute the expression (3.12) into the Maxwell equation to obtain an equation on $\mathcal{B}$:

$$\begin{aligned} d \star_4 d(f^{-1} X_I) = & -\frac{1}{6} C_{IJK} \Theta^J \wedge \Theta^K + 2\chi V_I f^{-1} G^- \wedge J^1 \\ & + 6\chi^2 f^{-2} (Q_{IJ} C^{JMN} V_M V_N + V_I X^J V_J) \eta_4. \end{aligned} \tag{3.17}$$

3.2 Near-horizon geometries

We are interested in classifying the near horizon geometries of supersymmetric black hole solutions of the above theory. The strategy is to introduce coordinates adapted to the presence of a Killing horizon, and then examine the conditions imposed by supersymmetry and the field equations in the near horizon limit, which we make precise below.

Following the reasoning of [14, 1], the line element of a supersymmetric black hole may be written in *Gaussian null coordinates* (v, r, x^a):

$$ds^2 = -r^2 \Delta(r,x)^2 dv^2 + 2dvdr + 2rh_a(r,x)dvdx^a + \gamma_{ab}(r,x)dx^a dx^b, \tag{3.18}$$

with the horizon located at $r = 0$. The supersymmetric Killing vector is $V = \partial/\partial v$ and for $r > 0$ (the exterior of the black hole) is timelike, so $f = r\Delta$ and thus $\Delta > 0$ for $r > 0$ (although we will allow for $\Delta = 0$ at $r = 0$). Spatial cross sections of the horizon are given by $r = 0$ and $v =$ constant: this defines a three-manifold, which we denote by H, with coordinates x^a. Since we are interested in black hole near-horizon geometries, ultimately we will require H to be compact. The *near-horizon limit* is defined by $r \to \epsilon r$ and $v \to v/\epsilon$ and $\epsilon \to 0$. After this limit is taken, the functions Δ, h_a, γ_{ab} in the line element (3.18) depend only on x^a (the coordinates on H). As in [1], first we will proceed by evaluating all equations as a power series in r and take the near-horizon limit at the end of this section. Let the volume form η_3 of γ_{ab} be chosen so that the spacetime volume form $\eta = dv \wedge dr \wedge \eta_3$ has positive orientation. Following [2] we work in a gauge where $i_V A^I = fX^I$. We can then write

$$A^I = r\Delta X^I dv + A^I_r dr + a^I_a dx^a. \tag{3.19}$$

Note that A^I_r does not survive the near-horizon limit. The two forms J^i may be written as [14]

$$J^i = dr \wedge Z^i + r(h \wedge Z^i - \Delta \star_3 Z^i) \tag{3.20}$$

where $\star_3$ is the Hodge dual with respect to γ_{ab} and the one-forms $Z^i = Z^i_a dx^a$ satisfy $\star_3 Z^i = \frac{1}{2}\epsilon_{ijk} Z^j \wedge Z^k$, i.e. they are orthonormal with respect to γ_{ab}. Substituting (3.20) into (3.10) yields

$$\begin{aligned} \hat{d}Z^i &= h \wedge Z^i - \Delta \star_3 Z^i + r\partial_r(h \wedge Z^i - \Delta \star_3 Z^i) \\ &\quad +3\chi\epsilon_{1ij}V_I \left[X^I \star_3 Z^j + a^I \wedge Z^j - rA_r^I(h \wedge Z^j - \Delta \star_3 Z^j)\right] \end{aligned} \tag{3.21}$$

and

$$\begin{aligned} \star_3\hat{d}h &= \Delta h + \hat{d}\Delta + r \star_3 (h \wedge \partial_r h) - r(\partial_r\Delta)h + 2r\Delta\partial_r h + r\Delta^2\epsilon_{ijk}Z^i\langle Z^j, \partial_r Z^k\rangle \\ &\quad +6\Delta\chi V_I \left(X^I + r\Delta A_r^I\right) Z^1. \end{aligned} \tag{3.22}$$

where $\hat{d}$ is the exterior derivative on H [1]. These equations closely resemble the minimal gauged case [1], the main difference being the presence of the scalar fields. To leading order in r the expressions become:

$$\hat{d}Z^i = h \wedge Z^i - \Delta \star_3 Z^i + 3\chi\epsilon_{1ij}V_I \left[X^I \star_3 Z^j + a^I \wedge Z^j\right] + \mathcal{O}(r) \tag{3.23}$$

and

$$\star_3 \hat{d}h = \Delta h + \hat{d}\Delta + 6\chi\Delta V_I X^I Z^1 + \mathcal{O}(r). \tag{3.24}$$

For $r > 0$, ω can be defined by $i_V\omega = 0$ and $d\omega = -d(f^{-2}V)$ so in this coordinate system,

$$\omega = -\frac{dr}{\Delta^2 r^2} - \frac{h}{\Delta^2 r}. \tag{3.25}$$

We next compute[12]

$$G^+ = \frac{1}{2}\left(f d\omega + i_V \star_5 d\omega\right), \tag{3.26}$$

which gives

$$G^+ = dr \wedge \mathcal{G} + r(h \wedge \mathcal{G} + \Delta \star_3 \mathcal{G}) \tag{3.27}$$

where

$$\mathcal{G} = -\frac{3\hat{d}\Delta}{2r\Delta^2} + \frac{3(\partial_r\Delta)h}{2\Delta^2} - \frac{3\partial_r h}{2\Delta} - \frac{1}{2}\epsilon_{ijk}Z^i\langle Z^j, \partial_r Z^k\rangle - \frac{3\chi V_I}{r\Delta}\left[X^I + \Delta r A_r^I\right] Z^1. \tag{3.28}$$

We now turn to the determination of the Maxwell fields (3.12). Following [18] write

$$\Theta^I = -\frac{2}{3}X^I G^+ + N^I \tag{3.29}$$

where $X_I N^I = 0$ and since N^I is self dual on the base it can be written as

$$N^I = dr \wedge T^I + r(h \wedge T^I + \Delta \star_3 T^I) \tag{3.30}$$

for some $T^I = T_a^I dx^a$. Substituting into (3.12), we find

$$\begin{aligned} F^I &= \left[\partial_r(r\Delta X^I)dr + r\hat{d}(\Delta X^I)\right] \wedge dv + dr \wedge Q^I + rh \wedge Q^I \\ &\quad +r\Delta \star_3 T^I - X^I \star_3 h - rX^I \star_3 \partial_r h - \frac{2}{3}rX^I\Delta \star_3 Z^i\epsilon_{ijk}\langle Z^j, \partial_r Z^k\rangle \\ &\quad +\chi V_J \star_3 Z^1 \left[9C^{IJK}X_K - 4X^I\left(X^J + r\Delta A_r^J\right)\right] \end{aligned} \tag{3.31}$$

[12] Note that $i_V \star_5 \lambda_p = f^{p-1} \star_4 \lambda_p$ for a p-form on the base manifold

where

$$Q^I = \frac{\hat{d}X^I}{r\Delta} - \frac{\partial_r X^I h}{\Delta} + \frac{1}{3}X^I Z^i \epsilon_{ijk}\langle Z^j, \partial_r Z^k\rangle + T^I$$
$$+\frac{1}{r\Delta}\chi V_J\left[2X^I(X^J + r\Delta A_r^J) - 9C^{IJK}X_K\right]Z^1. \tag{3.32}$$

Note that since we require F^I to be regular on the horizon, then so must Q^I be.[13] Reading off the the x^a components of F^I we get:

$$\begin{aligned}\hat{d}a^I &= rh\wedge Q^I + r\Delta \star_3 Q^I - \star_3\hat{d}X^I + r\partial_r X^I \star_3 h - r\Delta X^I\epsilon_{ijk}\star_3 Z^i\langle Z^j,\partial_r Z^k\rangle \\ &\quad -X^I\star_3 h - rX^I\star_3\partial_r h + 6\chi V_J\left[3C^{IJK}X_K - X^I(X^J + r\Delta A_r^J)\right]\star_3 Z^1 \\ &= -\star_3\hat{d}X^I - X^I\star_3 h + 6\chi V_J(3C^{IJK}X_K - X^I X^J)\star_3 Z^1 + \mathcal{O}(r)\end{aligned} \tag{3.33}$$

where the last equality follows from regularity of Q^I at $r = 0$. The Bianchi identity contracted with X_I implies

$$\hat{d}\star_3 h + \frac{2}{3}Q_{IJ}\hat{d}X^I\wedge\star_3\hat{d}X^J + 2\chi V_K X^K\hat{d}\star_3 Z^1 + 4\chi V_K\hat{d}X^K\wedge\star_3 Z^1 = \mathcal{O}(r). \tag{3.34}$$

In the ungauged case, $\chi = 0$ and therefore integrating (3.34) over H leads to $\hat{d}X^I = 0$, since Q_{IJ} is a positive definite metric on the scalar manifold [18]. However, in the gauged theory this conclusion cannot be drawn and indeed we will find explicit solutions where the scalars are *not* constant on H.

We next turn to the computation of the spin connection of γ_{ab} using (3.21), which allows us to deduce

$$\nabla_a Z_b^i = -\frac{\Delta}{2}(\star_3 Z^i)_{ab} + \gamma_{ab}(h\cdot Z^i + 3\chi V_I X^I\delta_i^1) - Z_a^i h_b - 3\chi V_I X^I Z_a^i Z_b^1 + 3\chi V_I a_a^I\epsilon_{1ij}Z_b^j + \mathcal{O}(r). \tag{3.35}$$

where ∇ is the connection of γ_{ab}. From (3.35) it is easy to deduce that

$$\star_3 d\star_3 Z^i = \nabla_b Z^{ib} = 2h\cdot Z^i + 6\chi V_I X^I\delta_1^i + 3\chi V_I\epsilon_{1ij}a^I\cdot Z^j + \mathcal{O}(r). \tag{3.36}$$

Finally, we can determine the Ricci tensor R_{ab} of γ_{ab}, using

$$R_{ab}Z^{ib} = \nabla_b\nabla_a Z^{ib} - \nabla_a\nabla_b Z^{ib}. \tag{3.37}$$

After an involved calculation, we find:

$$\begin{aligned}R_{ab} = &\left(\frac{\Delta^2}{2} + h^2 + 4\chi V_J X^J h\cdot Z^1 + \frac{2}{3}Q_{IJ}\hat{d}X^I\cdot\hat{d}X^J + 4\chi V_J Z^{1c}\partial_c X^I + 2\chi^2[6(V_I X^I)^2 - \mathcal{V}]\right)\gamma_{ab} \\ &-\nabla_{(a}h_{b)} - h_a h_b - 6\chi V_I X^I h_{(a}Z_{b)}^1 - 6\chi V_I\partial X_{(a}^I Z_{b)}^1 - 2\chi^2 Z_a^1 Z_b^1\left(9(V_I X^I)^2 - \mathcal{V}\right) + \mathcal{O}(r).\end{aligned} \tag{3.38}$$

[13]In minimal gauged supergravity it was found that the Maxwell field was automatically regular on the horizon [1]. With extra vector multiplets though, it seems we need this as an extra (reasonable) assumption — this was also found in the ungauged theory [18]. Note, however, $X_I F^I$ is automatically regular.

Note that all gauge dependent terms cancel, as they must. Our final task is to examine the Maxwell equation (3.17). We find that as in the minimal theory this imposes no new constraints. More precisely we find that the Maxwell equation leads to a second order equation for the scalars which can also be derived by taking the $\hat{d}$-derivative of (3.33) and using (3.34) and (3.36). There are no further conditions imposed by supersymmetry or the field equations.

We are interested in determining near-horizon solutions. Accordingly, as discussed in the beginning of this section, we shall henceforth work strictly in the near-horizon limit. This amounts to dropping all $\mathcal{O}(r)$ terms in the equations derived above and setting $r = 0$ in all other quantities. Therefore all equations are now defined purely on H and from now one we will denote the exterior derivative on H simply d.

3.3 Some general results

We have not been able to solve the near-horizon equations derived in the previous section in full generality. Indeed, it is not even known how to solve the near-horizon equations in full generality in minimal gauged supergravity [1, 10]. However, much like in the minimal case we have obtained some general results which allow one to classify near-horizon geometries into two classes: static ($V \wedge dV \equiv 0$) or non-static. The following lemmas provide conditions for this:

Lemma 1. The following conditions are equivalent: (a) $V \wedge dV \equiv 0$, (b) $dh = 0$, (c) $\Delta \equiv 0$.

Proof. Assume (a). The rab components of $V \wedge dV \equiv 0$ give $dh = 0$ so (a) implies (b). Now assume (b). If Δ is nonzero then equation (3.24) can be solved for Z^1 which leads to $Z^1 \wedge dZ^1 = 0$. Then equation (3.23) implies $Z^1 \wedge \star_3 Z^1 = 0$ which contradicts Z^1 having unit norm. Therefore (b) implies (c). Finally assume (c). Equation (3.24) implies $dh \equiv 0$. But $\Delta \equiv 0$ and $dh \equiv 0$ implies $V \wedge dV \equiv 0$. Therefore (c) implies (a).

Lemma 2. If Δ vanishes at a point then Δ vanishes everywhere.

Proof. Taking the divergence of (3.24) and using (3.34) and (3.35) gives:

$$\begin{aligned}\nabla^2\Delta = &-\left(h + 6\chi V_I X^I Z^1\right) \cdot \nabla\Delta \\ &+ \left[\frac{2}{3}Q_{IJ}\partial X^I \cdot \partial X^J - 2\chi V_I \partial X^I \cdot Z^1 - 8\chi V_I X^I (h \cdot Z^1 + 3\chi V_I X^I)\right]\Delta \end{aligned} \quad (3.39)$$

and therefore one sees that this equation for Δ is of the same form as in the minimal case. This allows one to repeat the argument used in [10] to prove that if Δ vanishes at p then all derivatives of Δ at p also vanish. Hence by analycity we deduce that $\Delta \equiv 0$.
We should point out that lemma 2 will not be used to derive any of the results in this paper and thus in particular we will not need to assume analycity on the horizon.

3.3.1 A special case

There is a special case in which the near-horizon equations can be solved, without the assumption of rotational symmetries. This case is specified by: h Killing, $\Delta =$ constant (possibly zero), $X^I =$ constant and $h = -3\chi V_I X^I Z^1$. It is not obvious that these conditions are consistent, however it turns out they are. These conditions will arise later in our

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analysis of $\mathrm{U}(1)^2$-invariant near-horizon solutions and are the analogues of the assumptions made in the analysis for minimal gauged supergravity [1]. The steps turn out to parallel the minimal case [1] very closely. Define $W = Z^2 + iZ^3$. In general, with no assumptions, equation (3.23) implies:

$$dW = \left[h + 3\chi V_I X^I - i\Delta Z^1 - 3\chi i V_I a^I\right] \wedge W \tag{3.40}$$

and thus $W \wedge dW = 0$. Therefore locally we can write $W = \sqrt{2} F dw$ for some complex functions F and w on H. Although w is gauge invariant, F is not and we choose to work in a gauge where F is real. As in [1], we can define real coordinates (x, y, z) by $w = (x+iy)/\sqrt{2}$ and $Z^1 = \partial/\partial z$. The metric in these coordinates reads:

$$ds_3^2 = (dz + \alpha)^2 + 2F^2 dw d\bar{w} \tag{3.41}$$

where $\alpha = \alpha_w dw + \alpha_{\bar{w}} d\bar{w}$ is a real one-form. So far we have not used any of our assumptions. From the assumptions it follows that Z^1 is Killing and thus α and F are independent of z. From the equation for dZ^1 (3.23) it follows that:

$$\partial_w \alpha_{\bar{w}} - \partial_{\bar{w}} \alpha_w = -i\Delta F^2 \tag{3.42}$$

just like in the minimal case. Thus we have determined α in terms of F (up to a gradient which can be absorbed into the definition of z). Equation (3.40) can be solved for $V_I a^I$ to get:

$$V_I a^I = -\frac{\Delta}{3\chi} Z^1 - \frac{i}{3\chi}\left(\frac{\partial_w F}{F} dw - \frac{\partial_{\bar{w}} F}{F} d\bar{w}\right) \tag{3.43}$$

and substituting this into (3.33) gives:

$$\partial_w \partial_{\bar{w}} \log F^2 = \left(2\chi^2 \mathcal{V} - 9\chi^2 (V_I X^I)^2 - \Delta^2\right) F^2 \tag{3.44}$$

which is Liouville's equation. There are three cases to consider depending on whether the r.h.s. is positive, zero or negative. Define $g^2\lambda \equiv -2\chi^2\mathcal{V} + 9\chi^2(V_I X^I)^2$. Importantly, λ can be positive, negative or vanish, depending on the values of the scalars. Note that in minimal supergravity $\lambda = -3$; we will see that it is possible to get some qualitatively different geometries by taking $\lambda \geq 0$ which have no counterpart in the minimal case. First let us deduce some conditions for which $\lambda \geq 0$. For simplicity work in $\mathrm{U}(1)^3$ supergravity in which case:[14]

$$\lambda(X^1, X^2, X^3) = -2(X^1X^2 + X^2X^3 + X^1X^3) + (X^1)^2 + (X^2)^2 + (X^3)^2. \tag{3.45}$$

It has the property that if $\sqrt{X^1} + \sqrt{X^2} \leq \sqrt{X^3}$ (or permutations of (123)) then $\lambda \geq 0$.

One can repeat the steps of [1] to get:

1. $\Delta^2 = -g^2\lambda$ which is only possible for scalars satisfying $\lambda \leq 0$. In this case Liouville's equation reduces to the wave equation. By a holomorphic change of coordinates it

[14] In the QFT literature this is known as the Källen function.

is then possible to set $F = 1$ and in these coordinates $\alpha = \frac{g\sqrt{-\lambda}}{2}(ydx - xdy)$, so the horizon becomes:

$$ds_3^2 = \left(dz + \frac{g\sqrt{-\lambda}}{2}(ydx - xdy)\right)^2 + dx^2 + dy^2 \qquad (3.46)$$

which for $\lambda < 0$ is the homogeneous metric on Nil and generalises the solution found in the minimal case [2]. However we can also take $\lambda = 0$ now, in which case the horizon is locally isometric to R^3. This solution has no counterpart in minimal gauged supergravity.

2. $\Delta^2 < -g^2\lambda$ which is only possible for $\lambda < 0$. Liouville's equation can be solved, and then a holomorphic change of coordinates leads to $F^2 = -\frac{1}{(g^2\lambda+\Delta^2)x^2}$ and $\alpha = -\frac{\Delta dy}{(g^2\lambda+\Delta^2)x}$. The geometry is the homogeneous metric on SL$(2, R)$:

$$ds_3^2 = \left(dz + \frac{\Delta}{|\Delta^2 + g^2\lambda|)}\frac{dy}{x}\right)^2 + \frac{1}{|\Delta^2 + g^2\lambda|}\left(\frac{dx^2 + dy^2}{x^2}\right). \qquad (3.47)$$

   Note that $\Delta = 0$ is allowed in this case: this corresponds to the horizon being locally isometric to $R \times H^2$.

3. $\Delta^2 > -g^2\lambda$ which is possible for any λ. After a homomorphic change of coordinates we have:

$$F^2 = \frac{2}{(\Delta^2 + g^2\lambda)(1 + w\bar{w})^2}, \qquad \alpha = -\frac{i\Delta}{(\Delta^2 + g^2\lambda)(1 + w\bar{w})}\left(\frac{dw}{w} - \frac{d\bar{w}}{\bar{w}}\right). \quad (3.48)$$

   Now introduce real coordinates via $w = \tan(\theta/2)e^{i\psi}$. For $\Delta > 0$ one can write:

$$ds_3^2 = \frac{1}{\Delta^2 + g^2\lambda}\left[\frac{\Delta^2}{\Delta^2 + g^2\lambda}(d\phi + \cos\theta d\psi)^2 + d\theta^2 + \sin^2\theta d\psi^2\right] \qquad (3.49)$$

   where $z = \Delta\phi/(\Delta^2 + g^2\lambda)$. This is the homogeneous geometry of a squashed S^3. The full near-horizon geometry is that of the black hole solutions found in [2].[15] One can also have $\Delta = 0$ when $\lambda > 0$; this gives:

$$ds_3^2 = dz^2 + \frac{1}{g^2\lambda}(d\theta^2 + \sin^2\theta d\phi^2) \qquad (3.50)$$

   which is locally isometric to $R \times S^2$. This solution has no counterpart in minimal supergravity.

This analysis has been purely local. We are mainly interested in a compact horizon H. It is easy to see that the $\lambda = 0$ case of 1 above gives a T^3 horizon, whereas the $\Delta = 0$ case of 3 gives an $S^1 \times S^2$ horizon.

[15] The near-horizon limit of their solution is parameterized by three independent constants called q_I, which are related to our parameters by $q_I = 4X_I/(\Delta^2 + g^2\lambda)$.

4. $U(1)^2$-invariant near-horizon geometries

Consider an asymptotically AdS_5 black hole admitting two rotational symmetries m_1 and m_2. The near-horizon solution will inherit these symmetries. Therefore we are interested in classifying all near-horizon solutions for which there are two commuting Killing fields m_1 and m_2 on H that preserve h, Δ, the Maxwell fields F^I and the scalars X^I.

Under these conditions a welcome simplification occurs for the Maxwell fields. A standard argument, which uses the Bianchi identity for F^I as well as the fact that the Lie derivatives of F^I along the Killing fields vanish, tells us that $F^I_{\mu\nu} m_1^\mu m_2^\nu$ is a constant. Since we are looking for solutions which are asymptotically globally AdS_5, both Killing fields vanish at a (different) point. Therefore $F^I_{\mu\nu} m_1^\mu m_2^\nu = 0$. This condition is inherited in the near-horizon limit.

We can choose local coordinates $x^a = (\rho, x^i)$ with $\partial/\partial x^i$ Killing, so that the metric on H is:

$$\gamma_{ab} = d\rho^2 + \gamma_{ij}(\rho) dx^i dx^j \tag{4.1}$$

and Δ, X^I and the components of h and F^I are functions only of ρ. We will allow $\partial/\partial x^i$ to be arbitrary linear combinations of m_i and thus they need not have closed orbits. We will enforce the fact that m_i have closed orbits once we have determined the local form of the solution.

We will define a positive function $\Gamma(\rho)$ and a one-form $k_i(\rho)dx^i$ by:

$$h = \Gamma^{-1} k_i dx^i - \frac{\Gamma'}{\Gamma} d\rho \tag{4.2}$$

where a prime denotes a derivative with respect to ρ. The components of the Maxwell fields on H can be written as:

$$\frac{1}{2} F^I_{ab} dx^a \wedge dx^b = B^I_i(\rho) d\rho \wedge dx^i \tag{4.3}$$

where we have used the fact that $F^I_{ij} = 0$ argued above.

Taking the dual of equation (3.33) and using (4.3) leads to:

$$\star_2 B^I = (X^I)' d\rho + X^I h - 6\chi V_J (3C^{IJK} X_K - X^I X^J) Z^1 \tag{4.4}$$

where $\star_2$ denotes the Hodge star with respect to the two-dimensional metric γ_{ij} (with volume form η_2 such that $\eta_3 = d\rho \wedge \eta_2$). Contracting with X_I implies

$$Z^1 = \frac{1}{2\chi V_I X^I} (\star_2 B - h) \tag{4.5}$$

where for convenience we have defined $B \equiv X_I B^I$. Now we can read off the ρ and i components of (4.4); the ρ component leads to

$$X^{I\prime} + \frac{2\Gamma'}{\Gamma} X^I - \frac{9C^{IJK} V_J X_K}{V_L X^L} \frac{\Gamma'}{\Gamma} = 0 \tag{4.6}$$

whereas the i component leads to an expression for B^I in terms of B. Contracting with V_I gives:

$$(V_I X^I)' + \frac{2\Gamma' V_I X^I}{\Gamma} - \frac{\Gamma' \mathcal{V}}{3\Gamma V_I X^I} = 0. \tag{4.7}$$

Using (4.5) and (4.7), the ρi component of equation (3.38) simplifies considerably leaving:

$$0 = R_{\rho i} = -\frac{1}{2}\Gamma^{-1}\gamma_{ij}(k^j)' \tag{4.8}$$

and hence

$$k^i \equiv \gamma^{ij} k_i = \text{constant}. \tag{4.9}$$

Next, using (4.5), equation (3.24) gives:

$$\Delta' + \frac{2\Delta\Gamma'}{\Gamma} = 0, \tag{4.10}$$

$$(\Gamma^{-1}k)' + 2\Delta \star_2 (\Gamma^{-1}k) = -3\Delta B \tag{4.11}$$

where for a one form $\omega_i(\rho)dx^i$ we defined $\omega' = \omega_i'(\rho)dx^i$. Hence

$$\Delta = \frac{\Delta_0}{\Gamma^2} \tag{4.12}$$

where Δ_0 is a non-negative constant. Substituting (4.5) into (3.23) leads to:

$$\Gamma^{-1}k \wedge \star_2 B - \frac{\Delta\Gamma'}{\Gamma}\star_2 1 = 0, \tag{4.13}$$

$$\left[\frac{1}{V_I X^I}(\star_2 B - \Gamma^{-1}k)\right]' = \frac{\Delta}{V_I X^I}(B + \star_2 \Gamma^{-1}k) - \frac{1}{V_I X^I}\frac{\Gamma'}{\Gamma}\star_2 B. \tag{4.14}$$

Now let us turn to equation (3.35). The $\rho\rho$ component gives:

$$h \cdot Z^1 + 3\chi V_I X^I = \frac{1}{2\chi V_I X^I}\left[\frac{\Gamma''}{\Gamma} - \frac{\Gamma'^2}{2\Gamma^2} - \frac{\Gamma'}{\Gamma}\frac{(V_I X^I)'}{V_I X^I}\right] \tag{4.15}$$

and the ij component gives:

$$\frac{\Gamma'}{4\chi V_I X^I \Gamma}\gamma_{ij}' = -\frac{\Delta\Gamma'}{4\chi V_I X^I \Gamma}\sqrt{\gamma}\epsilon_{ij} + \gamma_{ij}(h \cdot Z^1 + 3\chi V_I X^I) - Z_i^1(h + 3\chi V_I X^I Z^1)_j. \tag{4.16}$$

There are two qualitatively distinct cases depending on whether Γ is a constant or not.

Constant Γ case. From equations (4.2) and (4.9) one can see that h must be Killing, equation (4.10) implies Δ = constant and equation (4.6) implies X^I = constant. Further equations (4.15) and (4.16) imply $h = -3\chi V_I X^I Z^1$. This turns out to be a very similar case to the one studied in the minimal case [1] which can be solved without the assumption of the $\mathrm{U}(1)^2$ symmetry. This is the special case we solved in section 3.3.1.

Non-constant Γ case. In the minimal theory we found that Γ was a more convenient coordinate than ρ on H [10]. Due to the presence of the scalar fields and scalar potential we will see that a better coordinate emerges, although Γ will still be useful. We first address solving the scalar equation (4.6). From this one can deduce an equation for X_I:

$$X_I' + \frac{\Gamma'}{\Gamma} X_I - \frac{\Gamma'}{\Gamma} \frac{V_I}{V_L X^L} = 0 \tag{4.17}$$

and therefore an equation for the scalar potential:

$$\mathcal{V}' + \frac{\Gamma'}{\Gamma} \mathcal{V} - \frac{6\xi^3 \Gamma'}{\Gamma V_L X^L} = 0. \tag{4.18}$$

Now define the positive function $x(\rho)$ by:

$$x \equiv \frac{\Gamma \mathcal{V}}{6\xi^2}. \tag{4.19}$$

The equation for the scalar potential can now be written as:

$$x' = \frac{\xi \Gamma'}{V_I X^I} \tag{4.20}$$

We will find that the function x turns out to be a better coordinate that Γ (observe that $x = \Gamma$ in the minimal limit). Indeed equation (4.7) may be written as:

$$\frac{d}{dx}(\Gamma^2 V_I X^I) = 2\xi x \tag{4.21}$$

which integrates to:

$$\Gamma^2 V_I X^I = \xi x^2 + \xi c_1 \tag{4.22}$$

where c_1 is some integration constant. Further, plugging (4.20) into (4.22) leads to a differential equation relating the two coordinates x and Γ which is easily integrated to:

$$\Gamma^3 = H(x) \equiv x^3 + 3c_1 x + c_2 \tag{4.23}$$

where c_2 is an integration constant and thus from (4.22) we have determined $V_I X^I$ as a function of x. Equation (4.17) can be written as:

$$\frac{d}{dx}(H^{1/3} X_I) = \frac{V_I}{\xi} \tag{4.24}$$

which then integrates to:

$$X_I = H(x)^{-1/3} \left(\frac{V_I x}{\xi} + K_I \right) \tag{4.25}$$

where K_I are integration constants. If one calculates $\mathcal{V}$ from this expression for X_I one learns that $C^{IJK} V_I V_J K_K = 0$. Further the constraint relating the scalars then tells us that:

$$c_1 = \frac{9}{2\xi} C^{IJK} V_I K_J K_K, \qquad c_2 = \frac{9}{2} C^{IJK} K_I K_J K_K. \tag{4.26}$$

Hence we have now fully determined the scalars in terms of x.

Since we are now assuming Γ (and hence x) is not a constant, we can derive a number of useful results from equations (4.15) and (4.16). Firstly (4.15) gives:

$$(k \cdot Z^1) = \frac{1}{2\chi V_I X^I}\left[\Gamma'' + \frac{\Gamma'^2}{2\Gamma} - \frac{\Gamma'(V_I X^I)'}{\Gamma V_I X^I}\right] - 3\chi V_I X^I \Gamma = \frac{\xi\chi}{\Gamma}\frac{dy}{dx} \tag{4.27}$$

where the last equality follows from (4.20) and we have defined

$$y = \frac{1}{4\xi^2\chi^2}\Gamma x'^2 - \Gamma^3. \tag{4.28}$$

Multiply (4.16) by γ^{ij} and use (4.15) to get:

$$\frac{\Gamma''}{\Gamma} = \frac{\Gamma'}{2\Gamma}(\log\gamma)' + \frac{\Gamma'}{\Gamma}\frac{(V_I X^I)'}{V_I X^I} \tag{4.29}$$

which integrates to:

$$\sqrt{\det \gamma_{ij}} = \beta^2 |x'| \tag{4.30}$$

using (4.20) and where β is a positive constant (chosen to match with the minimal limit [10]).

In order to make further progress it is now necessary to split the analysis into two cases: $\Delta_0 > 0$ and $\Delta_0 = 0$.

4.1 Non-static near-horizon geometry

From lemma 1 we see that this case corresponds to $\Delta_0 > 0$. We also see that the constants k^i cannot both vanish; if they did, then $dh = 0$ and hence by lemma 1 $\Delta = 0$, contradicting $\Delta_0 > 0$. Assume that Γ (and thus x) is not constant — we have already dealt with the Γ constant case.

Eliminating B between equations (4.11) and (4.13) leads to:

$$k^i(\Gamma^{-1}k_i)' = \frac{3\Delta_0^2\Gamma'}{\Gamma^4} \tag{4.31}$$

and since k^i are constants one can integrate this to get:

$$k^i k_i = C^2\Gamma - \frac{\Delta_0^2}{\Gamma^2} \tag{4.32}$$

where C is a positive constant. Now, contract (4.16) with $k^i k^j$ and use (4.32) to get:

$$(k \cdot Z^1)^2 = C^2\Gamma - \frac{\Delta_0^2}{\Gamma^2} - \frac{C^2 x'^2}{4\xi^2\chi^2\Gamma}. \tag{4.33}$$

Now, eliminate $k \cdot Z^1$ between equations (4.27) and (4.33) to get:

$$\left(\frac{dy}{dx}\right)^2 + \frac{C^2}{\xi^2\chi^2}y = -\frac{\Delta_0^2}{\xi^2\chi^2} \tag{4.34}$$

which integrates to

$$y = -\frac{C^2}{4\xi^2\chi^2}(x-\alpha_0)^2 - \frac{\Delta_0^2}{C^2}. \tag{4.35}$$

where α_0 is an integration constant. This implies:

$$x'^2 = \frac{4\xi^2\chi^2 P(x)}{\Gamma} \tag{4.36}$$

where

$$P(x) = H(x) - \frac{C^2}{4\xi^2\chi^2}(x-\alpha_0)^2 - \frac{\Delta_0^2}{C^2}. \tag{4.37}$$

Eliminate B between equations (4.11) and (4.5) to get:

$$Z^1 = \frac{1}{2\chi V_I X^I}\left[-\frac{1}{3\Delta}\star_2(\Gamma^{-1}k)' - \frac{1}{3}\Gamma^{-1}k + \frac{\Gamma'}{\Gamma}d\rho\right]. \tag{4.38}$$

Now we will use the $GL(2,R)$ freedom associated with the x^i coordinates to set $k^1 = 1$ and $k^2 = 0$. Note that equation (4.32) implies:

$$\gamma_{11} = C^2\Gamma - \frac{\Delta_0^2}{\Gamma^2}. \tag{4.39}$$

Plugging our expression for y back into (4.27) and then equating this to k^i times the i component of (4.38) gives the following ODE:

$$\frac{d}{d\Gamma}\left(\frac{\gamma_{12}}{\gamma_{11}}\right) = \frac{\Delta_0\beta^2}{[C^2\Gamma^3-\Delta_0^2]^2}\left(\frac{\mathcal{V}}{6\xi^2}(2C^2\Gamma^3+\Delta_0^2) - 3C^2\alpha_0\Gamma^2 - (C^2\Gamma^3-\Delta_0^2)\frac{\Gamma}{6\xi^2}\frac{d\mathcal{V}}{d\Gamma}\right) \tag{4.40}$$

where we have used (4.20) and (4.30). Thankfully this integrates in a similar way to the minimal case to give:

$$\frac{\gamma_{12}}{\gamma_{11}} = \frac{\Delta_0\beta^2(\alpha_0 - x)}{C^2\Gamma^3-\Delta_0^2} \tag{4.41}$$

plus some integration constant which we may set to zero using some of the remaining $GL(2,R)$ freedom. We may use the remaining freedom to set $\beta = 1$. Now using (4.30) allows us to deduce γ_{22}. Therefore we have completely determined the 2-metric γ_{ij} in terms of x. It is thus convenient to use x, rather than ρ as the 3rd coordinate on H; γ_{xx} can be deduced from (4.36). The full near-horizon geometry in the non-static case is now determined. The final step is to determine the field strengths. From (4.4) and (4.38) we can deduce the components of F^I on H:

$$\frac{1}{2}F^I_{ab}dx^a\wedge dx^b = d\left[H^{-1/3}X^I\left(\Delta_0 dx^1 + (x-\alpha_0)dx^2\right)\right] \tag{4.42}$$

The field strengths F^I are then fully determined from the gauge potentials (3.19) upon taking the near-horizon limit. We now summarise our results for the non-static case below:

Summary of non-static near-horizon solutions.

(i) If Γ (and hence x) are not constant:

$$\gamma_{ab}dx^a dx^b = \frac{H(x)^{1/3}dx^2}{4\xi^2\chi^2 P(x)} + H(x)^{-2/3}\left(C^2H(x)-\Delta_0^2\right)\left(dx^1 + \frac{\Delta_0\left(\alpha_0 - x\right)}{C^2H(x)-\Delta_0^2}dx^2\right)^2 + \frac{4\chi^2\xi^2H(x)^{1/3}P(x)}{C^2H(x)-\Delta_0^2}(dx^2)^2, \tag{4.43}$$

$$\Delta = \frac{\Delta_0}{H(x)^{2/3}}, \qquad k = \frac{\partial}{\partial x^1}, \qquad h = H(x)^{-1/3}k - \frac{H'(x)}{3H(x)}dx \tag{4.44}$$

$$X_I = H(x)^{-1/3}\left(\frac{V_I}{\xi}x + K_I\right) \tag{4.45}$$

$$A^I = \frac{X^I}{H^{1/3}}\left[\Delta_0\left(H^{-1/3}rdv + dx^1\right) + (x-\alpha_0)dx^2\right] \tag{4.46}$$

where $H(x) = x^3 + 3c_1x + c_2$ and $P(x) = H(x) - \frac{C^2}{4\xi^2\chi^2}\left(x-\alpha_0\right)^2 - \frac{\Delta_0^2}{C^2}$ and C, Δ_0 are positive constants, α_0 an arbitrary constant and $c_1 = \frac{9}{2\xi}C^{IJK}V_IK_IK_K$, $c_2 = \frac{9}{2}C^{IJK}K_IK_JK_K$ where K_I are constants satisfying $C^{IJK}V_IV_JK_K = 0$.

(ii) If Γ is a constant, the scalars are a constant, Δ is a constant and the metric on H must be one of: the homogeneous metrics on the group manifold Nil, SL$(2,R)$ or squashed S^3 depending on the value of Δ, see section 3.3.1. The latter case arises as the near-horizon limit of the asymptotically AdS black hole solutions found in [2], as explained in section 3.3.1.

The near-horizon geometry of the supersymmetric black holes of [4] is non-static with non-constant Γ and hence must be described by (i). We will prove this below.

4.2 Static near-horizon geometry

Lemma 1 tells us this corresponds to $\Delta_0 = 0$. We will now analyse the Γ not constant case, as we dealt with the Γ constant case earlier. It is now possible to have $k^i = 0$. This is dealt with in the appendix.

Thus, now assume that k^i are not both zero. Equation (4.11) implies $\Gamma^{-1}k_i$ are constants. Since k^i are constants we can define a positive constant C by $C^2 = \Gamma^{-1}k_ik^i$. Use the $GL(2,R)$ freedom to set $k^1 = 1$ and $k^2 = 0$. Therefore γ_{11} and γ_{12} are both a constant times Γ and hence we can use some of the remaining $GL(2,R)$ freedom to set $\gamma_{12} = 0$ and therefore $\gamma_{11} = k^ik_i = C^2\Gamma$. We can now repeat some of the steps used in the non-static case. Namely, contract (4.16) with k^ik^j and use (4.32) to get:

$$(k\cdot Z^1)^2 = C^2\Gamma - \frac{C^2x'^2}{4\xi^2\chi^2\Gamma}. \tag{4.47}$$

Now, eliminate $k\cdot Z^1$ between equations (4.27) and (4.47) to get:

$$\left(\frac{dy}{dx}\right)^2 + \frac{C^2}{\xi^2\chi^2}y = 0 \tag{4.48}$$

which integrates[16] to

$$y = -\frac{C^2}{4\xi^2\chi^2}(x-\alpha_0)^2 . \tag{4.49}$$

where α_0 is an integration constant. This implies:

$$x'^2 = \frac{4\xi^2\chi^2 P(x)}{\Gamma} \tag{4.50}$$

where

$$P(x) = H(x) - \frac{C^2}{4\xi^2\chi^2}(x-\alpha_0)^2 . \tag{4.51}$$

Observe that all these equations can be obtained from the $\Delta_0 \to 0$ limit of the corresponding equations in the $\Delta_0 > 0$ case. We can now use (4.30) to deduce γ_{22} and hence have fully determined the near-horizon geometry in this case. It remains to deduce the Maxwell fields. Observe that equation (4.13) implies that $\star_2 B \propto k$ and thus $Z^1_i \propto k_i$. Using the solution for y and substituting into (4.27) implies that:

$$Z^1_i = -\frac{(x-\alpha_0)k_i}{2\chi\xi\Gamma^2} \tag{4.52}$$

and substituting this into (4.4) leads to the components of the Maxwell field on H:

$$\frac{1}{2}F^I_{ab}dx^a \wedge dx^b = d[H(x)^{-1/3}X^I(x-\alpha_0)dx^2]. \tag{4.53}$$

Thus we see that the components of the field strengths on H can also be obtained as the $\Delta_0 \to 0$ limit of the non-static case. We now summarise the static-near horizon solutions:

Summary of static near-horizon solutions:

(i) If Γ is a constant, the scalars are constants. H is either locally isometric to $R \times H^2$ ($\lambda < 0$), R^3 ($\lambda = 0$) or $R \times S^2$ ($\lambda > 0$); see section (3.3.1). The first is the near-horizon limit of a supersymmetric black "string" [19].

(ii) Γ (and hence x) are not constant and k^i not both zero. This leads to a solution which can be obtained by taking the $\Delta_0 \to 0$ limit of (4.43), which amounts to simply setting $\Delta_0 = 0$ in the solution (4.43).

(iii) Γ not constant, $k^i = 0$. This case is analysed in the appendix, where the local form of the solution is given. It contains a case with zero Maxwell fields and non-constant scalars. In the case of zero Maxwell fields and constant scalars, H is locally isometric to H^3 and the near-horizon solution locally to AdS_5.

As discussed in section 3.3.1 and below, the $R \times S^2$ example in solution (i) above may be compactified to describe the near horizon of a regular supersymmetric black ring. We shall see that the solution (ii) also describes the near-horizon geometry of a supersymmetric black ring but suffers from a conical singularity.

[16]This equation has another solution: $y = 0$. However, it can be shown this implies $k = 0$, which is the case we consider in the appendix.

4.3 Global analysis

The preceding analysis has been entirely local. We are primarily interested in those solutions that arise from the near-horizon limit of black holes, and hence we must restrict attention to solutions for which H is compact. This turns out to be a strong constraint, as a compact three-dimensional manifold with $\mathrm{U}(1) \times \mathrm{U}(1)$ isometry must be homeomorphic to $T^3, S^1 \times S^2, S^3$, or a lens space [25].

Consider first the static near horizon solutions. The $R \times H^2$ possibility in (i) is immediately excluded since H^2 cannot be compactified without breaking the rotational symmetries. Furthermore, the subcase in (iii) which has zero Maxwell fields and constant scalars, has H locally isometric to H^3, and hence cannot be compactified without breaking the rotational symmetries. For the non-static geometries, (ii) is ruled out apart from the case where H is isometric to a homogeneously squashed S^3. As we have already explained in section 3.3.1, this solution is the same as the near-horizon limit of the black holes of [2].

Consider now the geometries that are locally R^3 or $R \times S^2$ in case (i) of the static solutions. It is clear that we may compactify these to yield compact horizons with T^3 and $S^1 \times S^2$ geometry respectively. We emphasize that these cases only occur for particular (constant) values of the scalars, and *cannot* exist in minimal gauged supergravity. Further there are no known black hole solutions with such horizon geometries.

Finally, consider the remaining possibilities, all of which have x non-constant. We first note that because $\partial/\partial x^1$ is a linear combination of m_1, m_2, its norm, γ_{11}, is a scalar invariant. By definition $x > 0$ and further $dx/d\Gamma > 0$. Hence for all cases, γ_{11} is a monotonically increasing function of x. Therefore x is uniquely determined in terms of γ_{11} and is a globally defined function on H. Compactness of H then implies that x must achieve a distinct minima and maxima on H. Hence the one-form dx must vanish at two different positive values of x. Computing $(dx)^2$ for the near horizons with non-constant x, we find this excludes case (iii) (see appendix), leaving (i) and (ii) of the non-static and static solutions respectively as the only possibilities. We conclude any solution with compact H and non-constant x must be given by (4.43), whether non static ($\Delta_0 > 0$) or static ($\Delta_0 = 0$).

For these solutions,

$$(dx)^2 = \frac{4\chi^2\xi^2 P(x)}{\Gamma}, \tag{4.54}$$

so we must impose that $P(x)$ be non-negative and have at least two distinct positive roots x_1, x_2 with $P(x) > 0$ for $x_1 < x < x_2$. It is straightforward to see that this implies $P(x)$ must have another distinct positive root x_3 such that $x_1 < x_2 < x_3$ (compactness excludes $x_2 = x_3$). These conditions constrain the parameters of the solution. For example, the positivity of the scalars X_I (4.45) then tell us that $K_I > -V_I x_1/\xi$. Furthermore, note that x is defined only up to a multiplicative constant. Hence, one of the parameters of (4.43) may be removed by a suitable rescaling of x. Explicitly, for some constant $\Omega > 0$, (4.43) is invariant under

$$x \to \Omega x, \quad x^1 \to \Omega^{-1} x^1, \quad K_I \to \Omega K_I, \quad C^2 \to \Omega C^2, \quad \Delta_0 \to \Omega^2 \Delta_0, \quad \alpha_0 \to \Omega \alpha_0 \tag{4.55}$$

We now turn to a detailed analysis of these two cases.

4.3.1 $\Delta_0 = 0$: Unbalanced black ring

From (4.43) we can read off the metric on H:

$$\gamma_{ab}dx^a dx^b = \frac{H(x)^{1/3}dx^2}{4\xi^2\chi^2 P(x)} + C^2 H(x)^{1/3}(dx^1)^2 + \frac{4\xi^2\chi^2 P(x)}{C^2 H(x)^{2/3}}(dx^2)^2. \tag{4.56}$$

It is clear we must remove the conical singularities at $x = x_1$ and $x = x_2$ at which the Killing field $\partial/\partial x^2$ vanishes. If this were possible, by suitably fixing the period of x^2, then the metric (4.56) would describe a regular horizon with topology $S^1 \times S^2$, with S^1 and S^2 parameterised by x^1 and (x, x^2) respectively, i.e. the horizon of a supersymmetric black ring in AdS_5.

The condition for removing the conical singularities at x_1, x_2 in the compact 2-manifold covered by (x, x^2) is

$$\frac{H(x_2)}{H(x_1)} = \left(\frac{x_3 - x_2}{x_3 - x_1}\right)^2. \tag{4.57}$$

The r.h.s. is obviously less than unity. But since $H'(x) > 0$ (which follows from the fact $d\Gamma/dx > 0$), $H(x)$ is a monotonically increasing function of x and the l.h.s. of (4.57) is larger than one. So, although H has $S^1 \times S^2$ topology, it necessarily has a conical singularity at one of the poles of the S^2. If (4.56) represents the near horizon of a black ring, then that black ring would be unbalanced, i.e. require external forces to prevent it from self-collapse.

Let us consider the full five-dimensional geometry further. If we define a new radial coordinate $R = H(x)^{-1/3}r$, the spacetime metric is given by

$$ds^2 = H(x)^{1/3}\left[-C^2R^2dv^2 + 2dvdR + C^2\left(dx^1 + Rdv\right)^2\right] + \frac{H(x)^{1/3}dx^2}{4\xi^2\chi^2P(x)} + \frac{4\xi^2\chi^2P(x)}{C^2H(x)^{2/3}}(dx^2)^2 \tag{4.58}$$

The part of the metric in the square brackets is locally isometric to AdS_3. Therefore the full five dimensional metric is a warped product of AdS_3 with a squashed S^2 with a singularity at one of its poles. Note that in the limit of vanishing gauge coupling $\chi \to 0$ this solution reduces to $AdS_3 \times S^2$, the near-horizon geometry of an asymptotically flat supersymmetric black ring [21]. This near-horizon geometry can be oxidised on S^5 to 10d, where it can be made regular with a different topology. The horizon topology becomes $S^1 \times M_7$, where M_7 is some complicated compact manifold (see appendix). The special case $K_I = 0$ (which is a solution to the minimal theory) lifts to the solution of [29], which was shown to lead to a discrete set of regular warped AdS_3 geometries. In the appendix we outline how to do this for the more general case. However, these regular solutions cannot be reduced to 5d and therefore do not have an interpretation in terms of 5d black holes. Indeed, an interesting question is whether there exist 10d asymptotically $AdS_5 \times S^5$ black holes whose near-horizon geometry is given by these regular warped AdS_3 geometries.

4.3.2 $\Delta_0 > 0$: Topologically spherical black hole

The horizon metric is

$$\gamma_{ab}dx^a dx^b = \frac{H(x)^{1/3}dx^2}{4\xi^2\chi^2P(x)} + H(x)^{-2/3}\left[A(x)(dx^1 + \omega(x)dx^2)^2 + B(x)(dx^2)^2\right] \tag{4.59}$$

where

$$A(x) = C^2 \left(P(x) + \frac{C^2}{4\chi^2\xi^2}(x-\alpha_0)^2 \right) \qquad B(x) = \frac{4\chi^2\xi^2 H(x)P(x)}{A(x)} \qquad \omega(x) = \frac{\Delta_0(\alpha_0 - x)}{A(x)} \tag{4.60}$$

Recall $x_1 \leq x \leq x_2$. Note that $\gamma_{11} > 0$ unless $\alpha_0 = x_i$ where $i = 1, 2$. We will consider $\alpha_0 \neq x_i$ now and deal with the degenerate cases $\alpha_0 = x_i$ in the appendix — it turns out they can be obtained as the $\alpha_0 \to x_i$ limits of the generic case. The 2-metric γ_{ij} induced on surfaces of constant x is non-degenerate everywhere except at the endpoints $x = x_i$, where the Killing vectors $\omega(x_i)\partial_{x_1} - \partial_{x_2}$ vanish. Using $P(x_i) = 0$, we find

$$\omega(x_i) = \frac{4\chi^2\xi^2}{C^4(\alpha_0 - x_i)} \tag{4.61}$$

and thus $\omega(x_1) \neq \omega(x_2)$. This implies that the Killing field which vanishes at $x = x_1$ is distinct from the one which vanishes at $x = x_2$. To avoid conical singularities these Killing fields must generate rotational symmetries, i.e. have closed orbits. Accordingly, they must be proportional to the m_i and we write

$$m_i = -d_i \left(\omega(x_i) \frac{\partial}{\partial x^1} - \frac{\partial}{\partial x^2} \right) \tag{4.62}$$

for some constants d_i. Define coordinates ϕ_i such that $m_i = \partial/\partial\phi_i$ and $\phi_i \sim \phi_i + 2\pi$. The coordinate change from (x^1, x^2) to (ϕ_1, ϕ_2) is given by

$$x^i = -[\omega(x_1)d_1\phi_1 + \omega(x_2)d_2\phi_2], \qquad x^2 = d_1\phi_1 + d_2\phi_2. \tag{4.63}$$

Absence of conical singularities then fixes the constants d_i up to a sign:

$$d_i^2 = \frac{A(x_i)}{4\chi^4\xi^4 P'(x_i)^2}. \tag{4.64}$$

Note that using $P(x_i) = 0$ one can show that

$$A(x_i) = \frac{C^4(x_i - \alpha_0)^2}{4\chi^2\xi^2}. \tag{4.65}$$

The solution is now globally regular: H has S^3 topology with m_1 vanishing at x_1 and m_2 vanishing at x_2. In the appendix we show that the coordinate change from (x^1, x^2) to (ϕ_1, ϕ_2) is also valid in the special cases $\alpha_0 = x_i$ (although $d_i \to 0$ as $\alpha_0 \to x_i$, $d_i\omega(x_i)$ is non-zero in this limit). Therefore we need not treat this case separately anymore.

Relation to known black hole. Next we show that the near-horizon limit of the black holes of [4] are in fact isometric to the non-static solution we have derived in this paper (4.59). Observe that one can deduce the near-horizon limit of the black hole in [4] from section 2.7 of that paper. The near-horizon geometry we derived is parameterised by $(C^2, \alpha_0, \Delta_0, K_I)$ with one constraint $C^{IJK}V_JV_KK_I = 0$. We can rewrite this solution in terms of the parameters (x_1, x_2, x_3, K_I) which are related by

$$C^2 = 4g^2(x_1 + x_2 + x_3), \qquad \alpha_0 = \frac{x_1x_2 + x_1x_3 + x_2x_3 - 3c_1}{2(x_1 + x_2 + x_3)}, \tag{4.66}$$

$$\Delta_0^2 = C^2(x_1x_2x_3 + c_2) - \frac{C^4}{4g^2}\alpha_0^2. \tag{4.67}$$

Note that the roots x_i are not arbitrary positive real numbers such that $x_1 < x_2 < x_3$: they are further constrained by $\Delta_0^2 > 0$. Now, the scale transformation (4.55) can be used to ensure that $g(x_3 - x_1)/\Delta_0 < 1$. Use the remaining scale transformations on x (i.e. ones with $\Omega > 1$) to fix:

$$\frac{g}{\Delta_0}(x_1 + x_2 + x_3) = \left(1 + \sqrt{1 - \frac{g}{\Delta_0}(x_3 - x_2)} + \sqrt{1 - \frac{g}{\Delta_0}(x_3 - x_1)}\right)^2 \tag{4.68}$$

where the factors under the square roots are positive as a consequence of the ordering of the roots and $g(x_3 - x_1)/\Delta_0 < 1$. This allows one to define two positive constants a, b by:

$$\Xi_a \equiv 1 - a^2 g^2 = \frac{g(x_3 - x_2)}{\Delta_0}, \qquad \Xi_b \equiv 1 - b^2 g^2 = \frac{g(x_3 - x_1)}{\Delta_0} \tag{4.69}$$

so $0 < b < a < g^{-1}$. The equations (4.68) and (4.69) can be inverted:

$$x_1 = g\Delta_0\left(b^2 + \frac{2r_m^2}{3}\right), \qquad x_2 = g\Delta_0\left(a^2 + \frac{2r_m^2}{3}\right), \qquad x_3 = \frac{\Delta_0}{g}\left(1 + \frac{2g^2 r_m^2}{3}\right) \tag{4.70}$$

where $r_m^2 \equiv g^{-1}(a + b) + ab > 0$. Also define constant e_I:

$$e_I = \frac{2r_m^2}{3} + \frac{K_I}{g\Delta_0}, \tag{4.71}$$

and observe that the constraint on the K_I translates to $3\bar{X}^I e_I = 2r_m^2$ (as in [4]). Equation (4.71) implies:

$$c_1 = \frac{g^2\Delta_0^2}{3}\left(\beta_2 - \frac{4r_m^4}{3}\right), \qquad c_2 = g^3\Delta_0^3\left(\beta_3 - \frac{2r_m^2\beta_2}{3} + \frac{16r_m^6}{27}\right) \tag{4.72}$$

where $\beta_2 = \frac{27}{2}C^{IJK}\bar{X}_I e_J e_K$ and $\beta_3 = \frac{9}{2}C^{IJK}e_I e_J e_K$. Substituting equations (4.70) and (4.72) into (4.66) and (4.67) allows one to solve for Δ_0:

$$\Delta_0 = \frac{1}{2g^2\Xi_a\Xi_b\sqrt{\delta}} \tag{4.73}$$

where

$$\begin{aligned}\delta = \frac{1}{4g^2\Xi_a^2\Xi_b^2}&[4g^2(1 + ag + bg)^2\beta_3 - g^4\beta_2^2 + 2g^2(g^2a^2b^2 + a^2 + b^2)\beta_2 \\ &- g^2(g^2a^2b^2 - 2gab^2 - 2ga^2b + (a - b)^2)r_m^4]\end{aligned} \tag{4.74}$$

is the same δ appearing in [4]; note $\delta > 0$ is equivalent to $\Delta_0^2 > 0$. Thus we have determined Δ_0 in terms of a, b, e_I. This proves (looking at (4.70)) that the parameters x_1, x_2, x_3, K_I can be written uniquely in terms of a, b, e_I (this was not obvious as we defined a, b, e_I as functions of x_1, x_2, x_3, K_I). Now define a new coordinate θ by

$$\sin^2\theta = \frac{x_2 - x}{x_2 - x_1} \tag{4.75}$$

so the range $0 \le \theta \le \pi/2$ covers the entire range of x. The endpoints $x = x_1, x_2$ correspond to $\theta = \pi/2, 0$ respectively. This can be inverted:

$$x = x_1 \sin^2\theta + x_2 \cos^2\theta = g\Delta_0 \left(\rho(\theta)^2 + \frac{2r_m^2}{3} \right) \tag{4.76}$$

where $\rho(\theta) = a^2\cos^2\theta + b^2\sin^2\theta$ and the second equality follows from (4.70). This implies

$$P(x) = \Delta_0^3 g(a^2 - b^2)^2 \Delta_\theta \sin^2\theta\cos^2\theta, \qquad H(x) = g^3\Delta_0^3 F(\rho^2) \tag{4.77}$$

where $\Delta_\theta = 1 - g^2\rho(\theta)^2$ and $F(\rho^2) \equiv \rho^6 + 2r_m^2\rho^4 + \beta_2\rho^2 + \beta_3$. This leads to

$$F(\rho^2)^{1/3}\frac{d\theta^2}{\Delta_\theta} = \frac{H(x)^{1/3}dx^2}{4g^2P(x)}. \tag{4.78}$$

Note that this verifies that the $\theta\theta$ component of the near-horizon limit of the black hole in [4] is equal to the xx component of our non-static near-horizon metric. For completeness, we give the expressions for d_i and $\omega(x_i)$:

$$d_1 = \frac{2b^4g^2 + 4g^2b^2r_m^2 + g^2b^2a^2 + b^2 - a^2 + g^2\beta_2}{2g\Xi_b(a^2 - b^2)} \tag{4.79}$$

$$d_2 = -\frac{2a^4g^2 + 4g^2a^2r_m^2 + g^2b^2a^2 - b^2 + a^2 + g^2\beta_2}{2g\Xi_a(a^2 - b^2)} \tag{4.80}$$

$$\omega(x_1) = -\frac{1}{2g\Delta_0^3(4g^2b^2r_m^2 + 2b^4g^2 + g^2b^2a^2 + b^2 - a^2 + g^2\beta_2)(1 + ag + bg)^2} \tag{4.81}$$

$$\omega(x_2) = -\frac{1}{2g\Delta_0^3(4g^2a^2r_m^2 + 2g^2a^4 + g^2b^2a^2 + a^2 - b^2 + g^2\beta_2)(1 + ag + bg)^2}. \tag{4.82}$$

We have checked explicitly that our non-static near horizon solution (4.59) written in terms of the coordinates (θ, ϕ_1, ϕ_2) and the parameters (e_I, a, b) is exactly the same as the near horizon limit of the solutions [4] in their (θ, ψ, ϕ) notation, provided we identify $\phi_1 = \psi$ and $\phi_2 = \phi$.

5. Discussion

In this work, we have derived the most general near horizon geometry of a regular, supersymmetric asymptotically AdS_5 black hole solution of $\mathrm{U}(1)^3$ gauged supergravity with two rotational symmetries. We emphasize that all known five dimensional black hole solutions, including black rings, have two rotational symmetries. It has been suggested that stationary black holes with only one rotational symmetry could exist [14], essentially because the higher dimensional analogue of the rigidity theorem only guarantees this [30]. However, in this work we are interested in supersymmetric black holes.[17] In five dimensional ungauged supergravity the analysis is much simpler and one can in fact solve the near-horizon equations generally. It turns out that in that theory a supersymmetric near-horizon geometry must possess two rotational symmetries after all (in fact it must be a homogeneous space) [14, 18].

[17] The rigidity theorem [30] is only valid for non-extremal black holes.

The near-horizon geometries we derived all possess enhanced symmetry $SO(2,1) \times U(1)^2$ and are 1/2 BPS (from the point of view of minimal supergravity). The bosonic part of this is guaranteed on general grounds from [15] (in fact it follows from extremality rather than supersymmetry). However the supersymmetry of these backgrounds is also enhanced. We expect this to also hold on general grounds, although no proof of this has been given. A recent classification of 1/2 BPS geometries [31] revealed that only one $U(1)$ spatial isometry is guaranteed. Therefore the validity of our assumption of two rotational symmetries is still unclear.[18] It would be nice to lift this assumption from our analysis.

Our results, summarized in section 2, imply that the most general near horizon geometry of a topologically *spherical* black hole, with two rotational symmetries, is given by the near horizon geometry of the four-parameter black hole solutions of [4]. Therefore, any other topologically spherical supersymmetric black hole must either have fewer than two rotational symmetries, or have the same near horizon geometry (and hence the same horizon geometry) as the solution of [4]. Note that as the gauge coupling is turned off the black hole solution [4] reduces[19] to the BMPV black hole, which also depends on four parameters and has a topologically spherical horizon.

In contrast with the corresponding analysis in minimal gauged supergravity [10], we have been unable to rule out the existence of black holes with T^3 or $S^1 \times S^2$ horizons; these must have near-horizon geometries $AdS_3 \times T^2$ and $AdS_3 \times S^2$ respectively. These solutions, which have no analogue in the minimal theory, have constant scalars and can only occur in regions of this moduli space satisfying $\lambda(X^1, X^2, X^3) \geq 0$ where λ is given in (2.5).

An interesting question is whether a supersymmetric AdS_5 black ring actually exists in this theory. If there is such a solution, and it possesses two rotational symmetries, we have shown that it must have the near-horizon geometry $AdS_3 \times S^2$ (given by (2.10)), as discussed above. This solution is not generic in the sense that the constant scalars must take values such that $\lambda > 0$, which in particular excludes any such solutions from having equal charges (i.e. occuring in minimal gauged supergravity). As noted in section 2, it is a three parameter solution. It can be parameterised by (L, q^I) where L is the radius of the S^1 on the horizon and q^I are the dipole charges defined by

$$q^I = \frac{1}{4\pi} \int_{S^2} F^I. \tag{5.1}$$

For our near-horizon solution (2.10) it turns out that

$$\sum_{I=1}^{3} q^I = -\frac{1}{g}. \tag{5.2}$$

In contrast, the near horizon geometry of the analogous supersymmetric black ring of ungauged $U(1)^3$ supergravity, which can also be parameterised by an analogous set (L, q^I), has unconstrained dipoles and therefore four independent parameters. The radius of the

[18] Note that solutions to gauged supergravity with only one spatial $U(1)$ symmetry have been found in [32].

[19] To see this one must rescale the parameters of the black hole in an appropriate manner.

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S^2 in (2.10) is $(g\sqrt{\lambda})^{-1}$ and can be made small compared to the AdS radius g^{-1} (as λ can be large compared to 1). However, as one turns off the gauge coupling (and hence sends the AdS radius to infinity) its size will diverge. This assumes that one does not rescale the scalars (and hence gauge fields) by factors of g; indeed, when taking such "flat space" limits, usually one only considers possible rescalings of the parameters and coordinates of the solution and not the fields (e.g. as for the spherical black hole discussed above). Therefore, for these reasons, we conclude that if (2.10) is indeed the near horizon of a supersymmetric black ring in AdS_5, it would not reduce (as one turns off the gauge coupling g) to the analogous solution in the ungauged theory.

Moreover, as in the minimal case [10], we found a near horizon geometry that is a warped product of AdS_3 with a squashed S^2 that suffers from a conical singularity at one of its poles. Unlike the above case, in the limit $g \to 0$, it *does* reduce to $AdS_3 \times S^2$, the near horizon of an asymptotically flat black ring. Therefore it seems natural to consider this as the generalisation of the near-horizon geometry of the black ring of ungauged supergravity, despite it being singular. This four-parameter solution could arise as the near horizon of a supersymmetric *unbalanced* black ring. If so this would mean that rotation and electromagnetic forces are not sufficient to counteract the gravitational self-attraction while simultaneously satisfying the BPS equality. However, this suggests that by increasing the angular momenta in such a way as to break the BPS condition, one might be able to balance this configuration and construct a non-supersymmetric black ring.

Acknowledgments

HKK and JL are supported by PPARC. We would like to thank Mukund Rangamani and Harvey Reall for comments on the draft.

A. $\Delta_0 = 0$ and $k = 0$

Here we deal with a special case $\Delta_0 = 0 = k^i$. Equation (4.15) becomes:

$$\frac{\Gamma''}{\Gamma} + \frac{\Gamma'^2}{2\Gamma^2} - \frac{\Gamma'(V_I X^I)'}{\Gamma V_I X^I} = 6\chi^2 (V_I X^I)^2 \tag{A.1}$$

and therefore we see that Γ cannot be a constant. Therefore we may use Γ as a coordinate and write (A.1) as:

$$\frac{d}{d\Gamma}\left(\frac{\Gamma\Gamma'^2}{4\chi^2(V_I X^I)^2} - \Gamma^3\right) = 0 \tag{A.2}$$

and integrate to get

$$\Gamma'^2 = \frac{4\chi^2(V_I X^I)^2 Q(\Gamma)}{\Gamma} \tag{A.3}$$

where $Q(\Gamma) = \Gamma^3 - \Gamma_0^3$ and Γ_0 is a real integration constant. Integrating (4.14) we find

$$\star_2 B = \frac{V_I X^I \omega}{\xi\Gamma} \tag{A.4}$$

where ω_i are constants (possibly zero). Since Z^1 has unit norm, we must have

$$\omega_i \omega^i = \frac{4\chi^2 \xi^2 \Gamma_0^3}{\Gamma} \tag{A.5}$$

and thus $\omega_i = 0$ if and only if $\Gamma_0 = 0$. Now, equation (4.16) simplifies to

$$\frac{\Gamma'}{4\chi V_I X^I \Gamma} \gamma'_{ij} = \chi V_I X^I \left(1 + \frac{2\Gamma_0^3}{\Gamma^3}\right) \gamma_{ij} - \frac{3 V_I X^I}{4\chi \xi^2 \Gamma^2} \omega_i \omega_j. \tag{A.6}$$

Now we can perform a $GL(2,R)$ transformation on the coordinates x^i to set $\omega_2 = 0$ (this is of course trivial if $\omega_i = 0$). This equation then simplifies to:

$$\frac{d}{d\Gamma} \log \gamma_{i2} = \frac{Q'(\Gamma)}{Q(\Gamma)} - \frac{2}{\Gamma} \tag{A.7}$$

and thus γ_{12} and γ_{22} are the same function up to a multiplicative constant. Hence we can use more of the $GL(2,R)$ freedom to set $\gamma_{12} = 0$ and solve for $\gamma_{22} = Q(\Gamma)/\Gamma^2$. Now multiplying (A.6) by γ^{ij} gives:

$$\frac{d}{d\Gamma} \log \gamma = \frac{Q'(\Gamma)}{Q(\Gamma)} - \frac{1}{\Gamma} \tag{A.8}$$

and hence $\gamma = C^2 Q(\Gamma)/\Gamma$ for some constant $C > 0$. Thus we have determined the local form of the metric; converting to the x coordinate introduced in the main text gives:

$$ds_3^2 = \frac{H(x)^{1/3} dx^2}{4\chi^2 \xi^2 Q(x)} + C^2 H(x)^{1/3} (dx^1)^2 + \frac{Q(x)}{H(x)^{2/3}} (dx^2)^2 \tag{A.9}$$

where $Q(x) = H(x) - \Gamma_0^3$.

As discussed in the main text, for a compact horizon the one-form dx must vanish at two distinct points (if x is non-constant which is the case here). However, we have

$$(dx)^2 = \frac{4\chi^2 \xi^2 Q(x)}{H^{1/3}} \tag{A.10}$$

and hence dx can vanish at most at one point (for $\Gamma_0 > 0$ this point is defined by $\Gamma = \Gamma_0$; for $\Gamma_0 = 0$ there is no such point). Therefore (A.9) cannot correspond to a compact horizon. If $K_I = 0$ (in which case the scalars are constants) and $\omega_i = 0$ (in which case the Maxwell fields vanish) the horizon metric is locally isometric to hyperbolic space H^3, and the full near horizon geometry is then locally AdS_5.

B. The cases $\boldsymbol{\alpha_0 = x_i}$

Here we perform a global analysis of the cases $\Delta_0 > 0$ and $\alpha_0 = x_i$. Consider the horizon metric (4.59). For clarity first consider $\alpha_0 = x_1$. Note that in this case $A(x)$ vanishes at x_1 and is positive otherwise, and $B(x)$ vanishes only at x_2 and is positive otherwise. It follows that the 2-metric γ_{ij} induced on surfaces of constant x is non-degenerate everywhere on the interval $x_1 \leq x \leq x_2$ except at $x = x_1$, where the Killing field $\partial/\partial x^1$ vanishes, and at

$x = x_2$, where the Killing field $\omega(x_2)\partial/\partial x^1 - \partial/\partial x^2$ vanishes. Hence these Killing fields describe rotational symmetries and must have closed orbits. Accordingly they must be proportional to the m_i and we may write

$$m_1 = c_1 \frac{\partial}{\partial x^1}, \qquad m_2 = -d_2\left(\omega(x_1)\frac{\partial}{\partial x^1} - \frac{\partial}{\partial x^2}\right) \tag{B.1}$$

where c_1, d_2 chosen so that $m_i = \partial/\partial\phi_i$ and $\phi \sim \phi + 2\pi$. Removing the conical singularity at $x = x_1$ we find

$$c_1^2 = \frac{\Delta_0^2}{\chi^2\xi^2 C^4 H'(x_1)^2}. \tag{B.2}$$

Removing the conical singularity at $x = x_2$ gives

$$d_2^2 = \frac{C^4(x_2 - x_1)^2}{16\chi^6\xi^6 P'(x_2)^2} \tag{B.3}$$

The horizon metric is now globally regular and has S^3 topology with m_1 vanishing at the pole $x = x_1$ and m_2 vanishing at the other pole $x = x_2$. The coordinate change $(x^1, x^2) \to (\phi_1, \phi_2)$, given by

$$x^1 = c_1\phi_1 - d_2\omega(x_2)\phi_2 \qquad x^2 = d_2\phi_2, \tag{B.4}$$

may be obtained from (4.63) by taking the limit $\alpha_0 \to x_1$.

Note that the $\alpha_0 = x_2$ case is analogous and can also be made globally regular resulting in S^3 topology. One finds m_i vanishes at $x = x_i$ where now:

$$m_1 = -d_1\left(\omega(x_1)\frac{\partial}{\partial x^1} - \frac{\partial}{\partial x^2}\right), \qquad m_2 = c_2\frac{\partial}{\partial x^1} \tag{B.5}$$

and

$$d_1^2 = \frac{C^4(x_2 - x_1)^2}{16\chi^6\xi^6 P'(x_1)^2}, \qquad c_2^2 = \frac{\Delta_0^2}{\chi^2\xi^2 C^4 H'(x_2)^2} \tag{B.6}$$

and

$$x^1 = -d_1\omega(x_1) + \phi_1 c_2\phi_2, \qquad x^2 = c_2\phi_2 \tag{B.7}$$

which can be obtained from the $\alpha_0 \to x_1$ limit of (4.63).

C. 10d near-horizon geometries

The 5d near-horizon geometries we have derived in $\mathrm{U}(1)^3$ gauged supergravity can all be lifted to solution of type IIB supergravity using [26]:[20]

$$ds_{10}^2 = W^{1/2} ds_5^2 + W^{-1/2}\sum_{I=1}^{3}(X^I)^{-1}[d\mu_I^2 + \mu_I^2(d\varphi_I + gA^I)^2], \tag{C.1}$$

$$\begin{aligned} F_5 = (1+\star_{10})\Bigg(&2g\sum_{I=1}^{3}((X^I)^2\mu_I^2 - WX^I)\epsilon_5 - \frac{1}{2g}\sum_{I=1}^{3}(X^I)^{-1}\star_5 dX^I\wedge d\mu_I^2 \\ &+\frac{1}{2g^2}\sum_{I=1}^{3}(X^I)^{-2}d\mu_I^2\wedge(d\phi_I + gA^I)\wedge\star_5 F^I\Bigg), \end{aligned} \tag{C.2}$$

[20] Note that the X_i of [26] are equal to our X^I; the field strengths of [26] F^i are the same as ours F^I. Indeed the action (2.1) agrees with the one they give in [26] with the field identifications just described.

where $W = \sum_{I=1}^{3} \mu_I^2 X^I > 0$, and μ_I, φ_I are coordinates on S^5 such that $\sum_{I=1}^{3} \mu_I^2 = 1$.

It can occur that certain singular 5d solutions can actually be made regular when oxidised to 10d. This is indeed the case for the static near-horizon solution which we found (4.58). The 10d lift of this is

$$ds_{10}^2 = W^{1/2} H^{1/3} [ds^2(AdS_3) + ds^2(M_7)], \tag{C.3}$$

$$\begin{aligned} ds^2(M_7) &= \frac{dx^2}{4g^2 P(x)} + \frac{4g^2 P(x)}{C^2 H(x)} (dx^2)^2 \\ &\quad + \frac{1}{g^2 W H^{2/3}} \sum_{I=1}^{3} (x + 3K_I) \left[d\mu_I^2 + \mu_I^2 \left(d\varphi_I + \frac{g(x-\alpha_0)}{x+3K_I} dx^2 \right)^2 \right] \end{aligned} \tag{C.4}$$

where the AdS_3 satisfies $R_{\alpha\beta} = -(C^2/2) g_{\alpha\beta}$. This solution was also encountered in [27] and studied in the context of warped AdS_3 geometries. We should emphasise that any warped AdS_3 geometry, where the internal space is compact, can be thought of as a near-horizon geometry as can be seen by writing AdS_3 as in (4.58). Therefore we will now perform a global analysis of the above solution under the assumption that M_7 is compact. The resulting horizon is this case will be topologically $S^1 \times M_7$. Much of the analysis parallels that in [28]. The metric on M_7 has a local $\mathrm{U}(1)^4$ isometry. We will demand that M_7 is a compact manifold, and thus since we require the metric on M_7 to be globally regular we must have a global $\mathrm{U}(1)^4$ symmetry. Examining scalar invariants constructed from the global $\mathrm{U}(1)^4$ symmetry generators implies x is a globally defined function on M_7. Therefore as in 5d we must take $x_1 \leq x \leq x_2$ with $0 < x_1 < x_2 < x_3$ roots of $P(x)$. This metric on constant x slices is positive definite everywhere except at the points $x = x_1, x_2$ or $\mu_I = 0$ where it degenerates. The latter points are where $\partial/\partial\varphi_I$ vanish (as usual for an S^5) and the corresponding conical singularities can be removed by identifying $\varphi_I \sim \varphi_I + 2\pi$ as usual. Note that μ_I are globally defined functions since they are the norms of the globally defined $\partial/\partial\varphi_I$ Killing vectors. The points $x = x_1, x_2$ were also degenerate points in 5d. However the Killing vector which vanishes at these points in 10d is modified: instead

$$V_i = \frac{k_i}{2g} \left(\frac{\partial}{\partial x^2} - g(x_i - \alpha_0) \sum_{I=1}^{3} \frac{1}{x_i + 3K_I} \frac{\partial}{\partial \varphi_I} \right) \tag{C.5}$$

vanishes at $x = x_i$ where $i = 1, 2$, where k_i are constants. We see that these two Killing vectors are distinct: this is qualitatively different to what happens in 5d where it was the same Killing vector that vanished at these two points $(\partial/\partial x^2)$. In order to obtain a smooth metric these must generate rotational symmetries and thus near $x = x_i$ one can introduce coordinates $\chi_i \sim \chi_i + 2\pi$ such that $V_i = \partial/\partial\chi_i$. This is equivalent to removing the conical singularities at $x = x_i$ and will fix the constants k_i. For clarity we also introduce new coordinates $\Phi_I \sim \Phi_I + 2\pi$ such that $\partial/\partial\Phi_I = \partial/\partial\varphi_I$. The corresponding coordinate transformation appropriate to the degeneration points $x = x_i$ are $(x^2, \varphi_I) \to (\chi_i, \Phi_I)$ and defined by:

$$x^2 = \frac{k_i}{2g} \chi_i, \qquad \varphi_I = \Phi_I - C_i^I \chi_i \tag{C.6}$$

where

$$C_i^I = \frac{(x_i - \alpha_0)k_i}{2(x_i + 3K_I)}. \tag{C.7}$$

The absence of conical singularities at $x = x_i$ determines k_i^2 and thus k_i up to a sign. Without loss of generality we choose signs such that

$$k_i = \frac{C^2(x_i - \alpha_0)}{2g^2 P'(x_i)}. \tag{C.8}$$

Note that k_i and C_i^I are non vanishing since $H(x_i) > 0$. One can verify that:

$$\sum_{I=1}^{3} C_i^I = 1 + k_i \tag{C.9}$$

Since the *five* Killing vectors $V_i, \partial/\partial\varphi_I$ span a *four* dimensional vector space with basis $\partial/\partial x^2, \partial/\partial\varphi_I$ they must be linearly dependent. In order to avoid identifying arbitrarily close points the dependency must take the form:

$$\sum_{i=1}^{2} n_i V_i + \sum_{I=1}^{3} m_I \frac{\partial}{\partial\varphi_I} = 0 \tag{C.10}$$

for integers n_i, m_I which we may assume to be coprime. It is easy to see that any subset of four of these five integers must also be coprime. Therefore we must consider the constraint on the parameters of the solution imposed by (C.10); this is:

$$\sum_{i=1}^{2} k_i n_i = 0, \qquad m_I = \sum_{i=1}^{2} C_i^I n_i. \tag{C.11}$$

Note that (C.9) then implies

$$\sum_{I=1}^{3} m_I = n_1 + n_2, \tag{C.12}$$

which allows us to write m_3 in terms of the four other integers. Therefore in order to obtain a smooth metric in 10d, the parameters of the solution (C^2, α_0, K_I) need to be chosen such that (C.11) is satisfied. In principle one can invert these relations to obtain a metric parameterised by four integers n_1, n_2, m_1, m_2. In general, however, it seems possible only to find expressions in terms of these integers implicitly.

A special case in which it is straightforward to give explicit formulas for these inversions, is $K_I = 0$ for $I = 1, 2, 3$ which corresponds to the AdS_3 geometry of [29]. In this case we have $3m = n_1 + n_2$. For simplicity we use the scaling symmetry to set $C = 2g$ and thus $P(x) = x^3 - (x - \alpha_0)^2$. In [10] it was shown that the existence of three positive distinct roots of P is equivalent to $0 < \alpha_0 < 4/27$ and that we may introduce the parameter b, which must satisfy $0 < b < 1$, such that

$$\alpha_0 = \frac{4(1-b^2)^2}{(b^2+3)^3}, \qquad x_1 = \frac{\alpha_0(b^2+3)}{4} \qquad x_2 = \frac{\alpha_0(b^2+3)}{(1+b)^2}, \qquad x_3 = \frac{\alpha_0(b^2+3)}{(1-b)^2}. \tag{C.13}$$

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Therefore:

$$k_1 = \frac{2(b^2+3)}{(b+3)(b-3)}, \qquad k_2 = -\frac{b^2+3}{b(b+3)}. \tag{C.14}$$

The condition $n_1k_1 + n_2k_2 = 0$ can then be solved to give:

$$b = \frac{3n_2}{n_2 - 2n_1}. \tag{C.15}$$

This implies

$$\alpha_0 = \frac{(2n_2 - n_1)^2(n_2 - 2n_1)^2(n_1 + n_2)^2}{27(n_1^2 + n_2^2 - n_2n_1)^3} \tag{C.16}$$

The fact that $0 < b < 1$ is equivalent to either: (i) $n_1 + n_2 > 0$ and $n_1 > 0, n_2 < 0$, or (ii) $n_1+n_2 < 0$ and $n_1 < 0, n_2 > 0$. For case (i) observe that defining $q = (n_1+n_2)/3 = m$ and $p = -n_2$ gives the positive integers p, q of [29]. For case (ii) $q = (n_1 + n_2)/3$ and $p = -n_1$.

We now turn to the five-form and examine its regularity in the general case. We must prove this is regular everywhere on M_7 and thus globally defined. Observe that $dx \wedge dx^2$ is a globally defined non-vanishing 2-form, since near the degeneration points $x = x_0, x_1$ this is proportional to the volume form of the associated local R^2. Therefore $\epsilon_5 = H^{1/3}C^{-1}\mathrm{vol}(AdS_3)\wedge dx\wedge dx^2$ is also globally defined, which takes care of the first term in (C.2). The second term in (C.2) contains $\star_5 dX^I$ and thus we need to consider $\star_5 dx$. One finds that $\star_5 dx = 4g^2C^{-1}P(x)dx^2 \wedge \mathrm{vol}(AdS_3)$ is also globally defined, since $P(x)dx^2$ is regular at the degeneration points $x = x_0, x_1$ (as can be seen in Cartesian coordinates on the associated R^2). For the final term in (C.2) we need to compute $\star_5 F^I$; this is globally defined since it looks like $f(x)\mathrm{vol}(AdS_3)$ for some regular function $f(x)$. Also note that as $x \to x_i$:

$$d\varphi_I + gA^I \sim d\Phi_I + \frac{k_i}{2(x_i + 3K_I)}(x - x_i)d\chi_i \tag{C.17}$$

and therefore, since $(x-x_i)d\chi_i$ is regular at $x = x_i$, we see that $d\mu_I^2 \wedge (d\varphi_I + gA^I)$ must be globally defined. This completes the proof that F_5 is regular everywhere on M_7 and thus globally defined.

Note that in order to ensure this is a good solution of string theory one must also show that the five form is appropriately quantized. We will not consider this here, as we are only concerned with proving existence of regular near-horizon geometries. As in the $K_I = 0$ case [29], however, we do not anticipate this will lead to further constraints on the integers n_1, n_2, m_1, m_2.

References

[1] J.B. Gutowski and H.S. Reall, *Supersymmetric AdS_5 black holes*, *JHEP* **02** (2004) 006 [hep-th/0401042].

[2] J.B. Gutowski and H.S. Reall, *General supersymmetric AdS_5 black holes*, *JHEP* **04** (2004) 048 [hep-th/0401129].

[3] Z.W. Chong, M. Cvetič, H. Lü and C.N. Pope, *General non-extremal rotating black holes in minimal five- dimensional gauged supergravity*, *Phys. Rev. Lett.* **95** (2005) 161301 [hep-th/0506029].

[4] H.K. Kunduri, J. Lucietti and H.S. Reall, *Supersymmetric multi-charge AdS_5 black holes*, *JHEP* **04** (2006) 036 [hep-th/0601156].

[5] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200]; S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109]; E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[6] J. Kinney, J.M. Maldacena, S. Minwalla and S. Raju, *An index for* 4 *dimensional super conformal theories*, *Commun. Math. Phys.* **275** (2007) 209 [hep-th/0510251].

[7] M. Berkooz, D. Reichmann and J. Simon, *AFermi surface model for large supersymmetric AdS_5 black holes*, *JHEP* **01** (2007) 048 [hep-th/0604023].

[8] R. Emparan and H.S. Reall, *A rotating black ring in five dimensions*, *Phys. Rev. Lett.* **88** (2002) 101101 [hep-th/0110260].

[9] J.P. Gauntlett and J.B. Gutowski, *All supersymmetric solutions of minimal gauged supergravity in five dimensions*, *Phys. Rev.* **D 68** (2003) 105009 [*Erratum ibid.* **D 70** (2004) 089901] [hep-th/0304064].

[10] H.K. Kunduri, J. Lucietti and H.S. Reall, *Do supersymmetric anti-de Sitter black rings exist?*, *JHEP* **02** (2007) 026 [hep-th/0611351].

[11] H. Lin, O. Lunin and J.M. Maldacena, *Bubbling AdS space and* 1/2 *BPS geometries*, *JHEP* **10** (2004) 025 [hep-th/0409174].

[12] B. Chen et al., *Bubbling AdS and droplet descriptions of BPS geometries in IIB supergravity*, *JHEP* **10** (2007) 003 [arXiv:0704.2233].

[13] M. Cvetič, H. Lü, C.N. Pope, A. Sadrzadeh and T.A. Tran, *Consistent* SO(6) *reduction of type IIB supergravity on S^5*, *Nucl. Phys.* **B 586** (2000) 275 [hep-th/0003103].

[14] H.S. Reall, *Higher dimensional black holes and supersymmetry*, *Phys. Rev.* **D 68** (2003) 024024 [*Erratum ibid.* **D 70** (2004) 089902] [hep-th/0211290].

[15] H.K. Kunduri, J. Lucietti and H.S. Reall, *Near-horizon symmetries of extremal black holes*, *Class. and Quant. Grav.* **24** (2007) 4169 [arXiv:0705.4214].

[16] D. Klemm and W.A. Sabra, *General (anti-)de Sitter black holes in five dimensions*, *JHEP* **02** (2001) 031 [hep-th/0011016].

[17] J.B. Gutowski and W. Sabra, *General supersymmetric solutions of five-dimensional supergravity*, *JHEP* **10** (2005) 039 [hep-th/0505185].

[18] J.B. Gutowski, *Uniqueness of five-dimensional supersymmetric black holes*, *JHEP* **08** (2004) 049 [hep-th/0404079].

[19] D. Klemm and W.A. Sabra, *Supersymmetry of black strings in $D = 5$ gauged supergravities*, *Phys. Rev.* **D 62** (2000) 024003 [hep-th/0001131].

[20] S.L. Cacciatori, D. Klemm and W.A. Sabra, *Supersymmetric domain walls and strings in $D = 5$ gauged supergravity coupled to vector multiplets*, *JHEP* **03** (2003) 023 [hep-th/0302218].

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[21] H. Elvang, R. Emparan, D. Mateos and H.S. Reall, *Supersymmetric black rings and three-charge supertubes*, *Phys. Rev.* **D 71** (2005) 024033 [hep-th/0408120].

[22] P. Davis, H.K. Kunduri and J. Lucietti, *Special symmetries of the charged Kerr-AdS black hole of $D = 5$ minimal gauged supergravity*, *Phys. Lett.* **B 628** (2005) 275 [hep-th/0508169].

[23] A. Sinha, J. Sonner and N.V. Suryanarayana, *At the horizon of a supersymmetric AdS_5 black hole: isometries and half-BPS giants*, *JHEP* **01** (2007) 087 [hep-th/0610002].

[24] A. Sinha and J. Sonner, *Black hole giants*, *JHEP* **08** (2007) 006 [arXiv:0705.0373].

[25] R.H. Gowdy, *Vacuum space-times with two parameter spacelike isometry groups and compact invariant hypersurfaces: topologies and boundary conditions*, *Ann. Phys. (NY)* **83** (1974) 203.

[26] M. Cvetič et al., *Embedding AdS black holes in ten and eleven dimensions*, *Nucl. Phys.* **B 558** (1999) 96 [hep-th/9903214].

[27] J.P. Gauntlett, N. Kim and D. Waldram, *Supersymmetric AdS_3, AdS_2 and bubble solutions*, *JHEP* **04** (2007) 005 [hep-th/0612253].

[28] M. Cvetič, H. Lü, D.N. Page and C.N. Pope, *New Einstein-sasaki spaces in five and higher dimensions*, *Phys. Rev. Lett.* **95** (2005) 071101 [hep-th/0504225].

[29] J.P. Gauntlett, O.A.P. Mac Conamhna, T. Mateos and D. Waldram, *Supersymmetric AdS_3 solutions of type IIB supergravity*, *Phys. Rev. Lett.* **97** (2006) 171601 [hep-th/0606221].

[30] S. Hollands, A. Ishibashi and R.M. Wald, *A higher dimensional stationary rotating black hole must be axisymmetric*, *Commun. Math. Phys.* **271** (2007) 699 [gr-qc/0605106].

[31] J.B. Gutowski and W.A. Sabra, *Half-supersymmetric solutions in five-dimensional supergravity*, arXiv:0706.3147.

[32] P. Figueras, C.A.R. Herdeiro and F. Paccetti Correia, *On a class of 4D Kähler bases and AdS_5 supersymmetric black holes*, *JHEP* **11** (2006) 036 [hep-th/0608201].